Geometric Analysis of the Mask Mosaic in Metropolis

Metropolis Mask Mozaiğinin Geometrik Çözümlemesi

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(Received 27 October 2015, accepted after revision 8 November 2016)

Abstract

The present article refers to the geometric analysis of the floor mosaics in a Reception Hall of the ancient city of Metropolis. In the course of this analysis a geometric plan of the mosaic frame was elaborated, while at the same time the geometric shapes and the geometric origin of some floral pattern were revealed and its standard drawing procedure was determined. Using a theoretical argument for the relation between the geometric plan and its execution as well as for the figures' construction order, preliminary assessments were performed before the measurement. As a consequence significant differences between the ratios of the planned and the measured lengths of the pavement could be detected. It was with the use of these ratios that an estimation of the mosaic sections' construction order was made. Furthermore, there were observed the reflection of mathematic knowledge in the mosaic, the relation between the repeated figures and the Pythagorean theorem as well as the relation between artistic functions and reasoning methods. It is obvious that mosaic artisans skilfully based their work on a profound knowledge of geometry. In this article is shown that through the geometric analysis of mosaics further scientific results within the fields of mosaic research, restoration and conservation, history of art, history of mathematics and archaeological studies in general can be achieved.

Keywords: *Metropolis, mosaic, mathematic, geometry, analysis.*

Öz

Bu çalışmada, Metropolis antik kentinde bulunan Resepsiyon Salonu taban mozaiği geometrik olarak analiz edilmiş ve geometrik planı ortaya çıkarılmıştır. Geometrik şekillerin yanı sıra, bazı bitkisel motiflerin de geometrik kökeni ortaya çıkarılmış ve standart çizim metodu belirlenmiştir. Geometrik plan ve uygulama arasındaki ilişkiler ve figürlerin yapılış sırası teorik olarak tartışılarak ölçüm öncesinde ön değerlendirmeler yapılmıştır. Figürlerin planlanan uzunlukları ile ölçülen uzunluklar arasında anlamlı orantılar bulunduğu tespit edilmiş ve bu oranlar kullanılarak, mozaik bölümlerinin inşa ediliş sırası tahmin edilmiştir. Matematik bilgisinin mozaik eser üzerindeki yansımaları, tekrarlanan figürlerin Pisagor teoremiyle ilişkisi ve sanat operasyonlarıyla akıl yürütme metotları arasındaki ilişki değerlendirilmiştir. Bu sayede mozaik sanatçılarının geometri bilgisini ustalıkla kullandıkları anlaşılmıştır. Mozaik eserler üzerinde yapılacak geometrik analizlerin mozaik araştırmaları, konservasyon çalışmaları, sanat, matematik tarihi ve arkeoloji çalışmaları için oldukça kullanışlı bilgiler üretme potansiyeline sahip olduğu sonucuna varılmıştır.

Anahtar Kelimeler: *Metropolis, mozaik, matematik, geometri, çözümleme.*

1. Introduction

The floor mosaics in the so-called Reception Hall in the ancient city of Metropolis (Fig. 1-2) have been chosen as the principal subject of this study. The need to proceed with the research and interpretation of the aforementioned mosaics arose as there are so many geometric patterns and human figures on the mosaics but no systematic surveys

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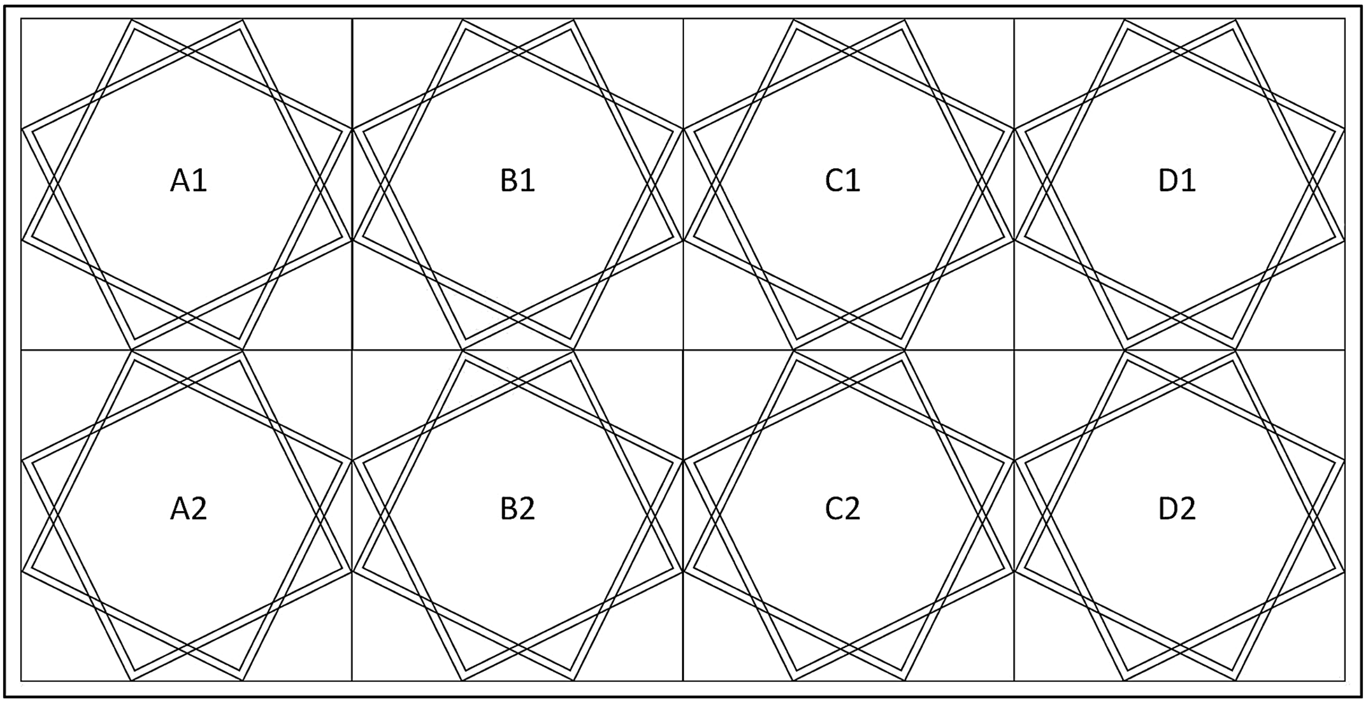


of the construction of geometric patterns. Furthermore, there was also a strong need to shed a light on the construction process of the mosaics that could lead the way to suitable conservation methods.

Figure 1, 2
Geometric floor mosaic in the Reception Hall of Metropolis (Öz 2012b)

It is quite difficult to study the geometric patterns in an appropriate chronological context, as the same pattern has been formed by different masters in different periods and used to decorate the interiors. The main objective is to prove that, especially in the Roman period, the applications used in the process of designing a mosaic were based not only on experience, but also on the knowledge of geometry. Therefore, the mosaic design method applied in the ancient period will be revealed by determining the dominant organization process in patterns, the element modules and the geometric rules used in organizing the whole of the mosaic. Another subject to be studied, as an important design principle, is the pattern diversification and the pattern expansion applications through their repetition in variable rhythms –which is considered to be a progress in decoration– achieved by the reassessment of the motif repetition.

The geometric analysis of mosaics includes examination of its mathematical plan and application methods, establishment of the connections with the construction process, interpretation and measurement of the results of this research in terms of archaeology, art and mathematics. At this point it is important to note that geometrical studies and methods do not cause any harm to the monuments.

1.1. Metropolis Mask Mosaic

Metropolis is an important ancient city located in the center of the Western Anatolia coast –which was named Ionia in the ancient period– between Smyrna and Ephesus. It has proven to be just as important as the rest of the cities in the same area, as the quality of its ruins and finds which came up during recent excavations shows. City's real establishment and development took place in the Hellenistic period as it was influenced by the Kingdom of Pergamon and enhanced significantly during the Roman period (Aybek et al. 2009: 39). Although there are technical and stylistic differences in each one of these mosaics, it can be generally assumed that the mosaic tradition in Metropolis was enriched during the 3rd and 4th centuries A.D. (Öz 2012a: 147).

The building, identified as the Reception Hall of the Theater because of its theatrical symbols (Meriç 1999: 336), is located on the east side of the Theatre and its floor is decorated with panel mosaics. In fact, the mosaic floor consists of two different panels, one at the center and one on the eastern border. The eastern panel (2,02 x 3,48 m sized) is important since it conveys an impression of a Reception Hall. Eight out of the eleven figures on this panel consist of bird and fish depictions, while the remaining three figures are theatre masks located in the center of the panel. This way the aforementioned symbols are related to both theatre and banquet. The annexes, which functioned as a cellar or a kitchen, highlight the function of the building. Similar examples of this building are found in the Terrace House of Ephesus (Lang-Auinger 1996: 205) and the Bau Z in Pergamon (Radt 1988: 102).

The middle panel is surrounded by a frame consisting of a 2,70 x 3,78 m sized, intertwined square, which is divided into six equal parts. There is one part in the southeastern area which is not preserved, but the other five parts reveal a very high quality workmanship with figural mosaics (Öz 2012b: 703). Many different ideas and suggestions have been presented for the interpretation regarding the motive and the dating of the five figures on the middle panel. According to Recep Meriç, who was the first person to excavate this place, the panels have figures relating to Dionysus (Meriç 2004: 99) or the Four Seasons (Parrish 2007: 19) and can be dated back to the 2nd – 3rd century A.D.

2. Geometric Analysis and Comparison

2.1. The Theoretical and the Real Measurements

The ABCD square, within which there are two square drawn, is the primary mosaics pattern and constitutes the geometric frame of the mosaics (Fig. 3). Other geometric figures, apart from the primary pattern's square trio, are incorporated naturally while at the same time bringing together the element forms successively. The square is considered to be one of the simplest concepts in mathematics. In fact, in every method including a set of application of geometry rules a square drawing procedure needs to be followed. First, a carrier square named ABCD square must be drawn and then there is a special drawing process of two more squares with their corner points on the carrier square. The accurate determination of those corner points is a delicate work, because smoothness of the octagon frames, which portraits will be placed into them, require the identity of this two squares.

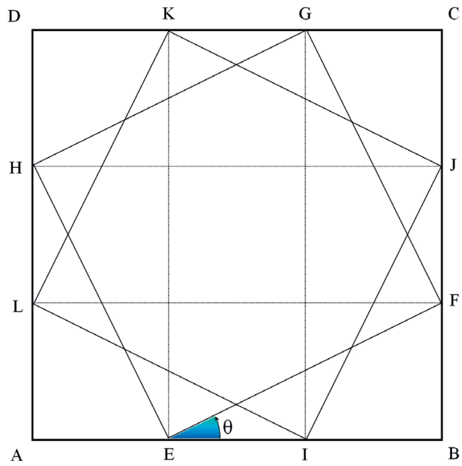


Figure 3
Primary pattern
of mosaic

In order for the design and construction phase of the primary pattern to be easier, a suitable method would be to divide each edge of the carrier frame into three equal parts or conversely to preconceive the edge length of the carrier frame as three units. Another method would be to determine the corner points of the inner squares with the use of a ruler or a compass. In the first method, the tanθ rate measured is approximately 1/2 (θ=26,6^0), while in the second method, the tanθ rate measured is different from 1/2 and there are no three equal parts on the edge of the carrier frame.

The ABCD square has been used repeatedly, but due to workmanship defects and variable sizes of little stones, deformation has occurred in the mosaic figures during tessellation process. In fact, the rate of this deformation may be used as an indication of the quality of workmanship. Actually, the section whose size is closer to the size of the element ABCD square, must be the part of the mosaics to be constructed in advance during the construction process, because as the construction progresses, deformation increases and the size of the square shaped parts of the mosaics diverges from its correct or original measurement. So patterns whose initial shape is square cease to be squares and turn into ordinary rectangular, with their 90 degrees angles increasing or decreasing.

There are eleven octagon frames in the mosaics. The smoothness of these frames depends on the smoothness of the ABCD square and the smoothness of the other two squares which constitute the content of the ABCD square. The octagon frames are actually the intersecting sides of these squares. If the squares are deformed, the octagons are also deformed. It should be noted that as much as an octagon is deformed chances are that this octagon was most recently constructed.

The edge lines of the ABCD square are the starting lines for the tessellation process. The stones are placed in rows from the edge lines towards the inner side of the square frame and if this process is done in an incorrect direction, then deformation occurs. There are dark stones used to give a shadow impression in-between the telescopic octagon patterns which actually show that the method been used is the parallel shifting method.

The vertices of the triangles are under the portraits, which indicates that the stones were laid in rows from the edge lines towards the inner side of the octagonal frame. On the other hand, stones were laid out right from the outline of the portraits. According to this tessellation logic, portraits cannot be placed into mosaics if there is not a geometric frame ready beforehand. Therefore, it can be concluded that portraits were most recently placed into the space allocated to them.

2.2. Identification of Deformation Theorem

When planned patterns are being built, deformations occur as a result of both labor and material. A deformation can be identified as the ratio between the product of measured lengths and planned measures. Deformation rates can be positive, negative or zero. Positive rates is an indication that planned lengths are greater than the measured lengths, whereas negative rates indicate that planned lengths are smaller than the measured ones. Zero rates is an indication that the plan was perfectly implemented, in other words that there is actually no existing deformation.

It has been observed that in repeat domains the carrier square of the primary pattern was been divided into approximately three equal parts. The lengths of these parts are concentrate around 26,00 cm. Consequently, the primary pattern should initially have been planned to be a 78,00 cm sized square pattern. Finally, the mosaic frame should have been a 156,00 - 312,00 cm sized frame and each edge of the octagons should have been a 19,30 cm sized edge. However, the short edge of the mosaics frame is measured to be a 149,00 cm sized edge. This incomplete measure is the first source of any deformation affecting all of the mosaics' square patterns which loose their square features in order to compensate for this incomplete measure.

The deformation rates of repeat domains differ from each other and that there is an internal consistency between the rates. These rates make it possible to compare between repeat domains. The domain with the smallest deformation rate among the repeat domains is the D2, as it is seen that the rates are steadily increasing from D2 to A1. The D2 domain is located in the corner of the venue.

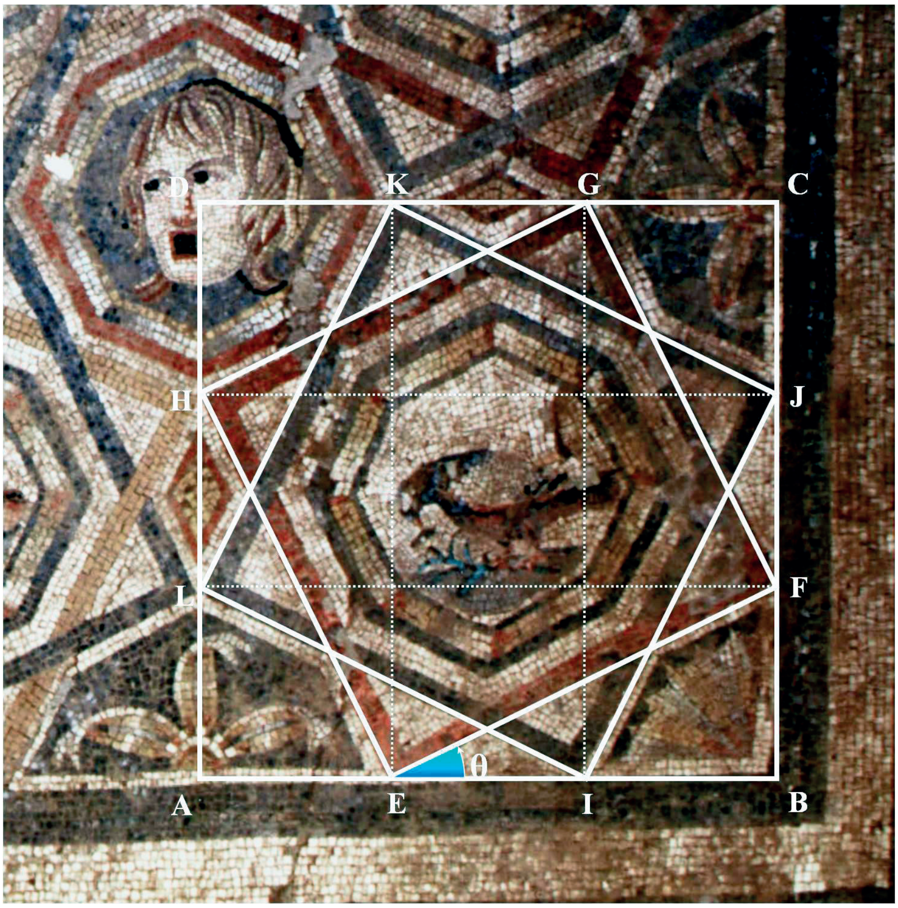
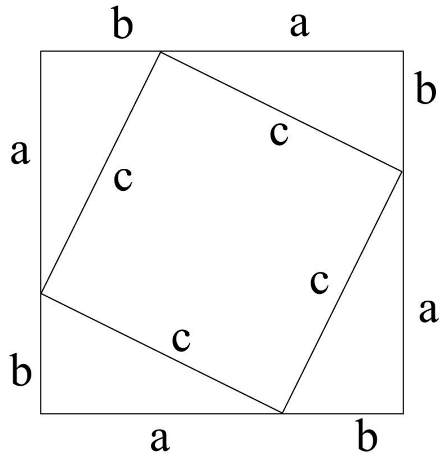


Figure 4
D2, first constructed
section of the mosaic panel

When geometric features and deformation rates are evaluated together, it can be understood that the D2 domain must have been the first section of the mask mosaic to be constructed (Fig. 4). Although there is a partial pattern absence in the A2 and A1 domains and the measurement process cannot be executed completely because of this absence, measurements made on surviving parts of lost patterns indicate that the domains C2, B2, A2, D1, C1, B1, A1 must have been constructed respectively after the D2 domain.

2.3. The Primary Pattern and the Mathematical Theorems

There are about 400 different proofs of the Pythagorean theorem. One of them is known as the Chou-pei Suan-ching proof, which came from China circa 250 B.C. (Lietzmann 1966; Nelson 1993; Veljan 1993: 259-272; Beryozin 1994: 25-28). As it can be seen the square duo figure of the primary pattern and the geometric figure used in the Chou-pei Suan-ching proof are the same (Fig. 5). The date of the proof figure precedes the construction date of the mosaic. This is an expected consistency, which leads to the conclusion that the proof figure had gained prominence in places far away from its source.



$$(a+b)^2 = c^2 + 4.\left(\frac{ab}{2}\right) \Rightarrow a^2 + b^2 = c^2$$

Figure 5
A mathematical proof of primary pattern (Lietzmann 1966)

The hypothesis of this paper is that there is a solid relation between the beauty concept and the mathematic rules. As it can be seen the mosaic patterns had been designed and constructed in accordance with a strict geometric plan and the mosaic artists had actually studied to be design engineers. This conclusion supports the idea that original portraits located in mosaic works could be made to adhere completely to a geometric plan and leads the way to studies aiming to examine mosaic portraits in terms of geometry.

One of the famous problems of the ancient era was known as the trisection of an angle with the use of only a measureless ruler and a compass (Heath 1921: 235). Because, according to the information provided by Vitruvius, mosaic masters were working using only ruler, compass and spirit level (Vitr. VII.I.3). The use of geometry knowledge in mosaic figures shows that the studies aiming to solve this ancient problem and determining the drawing facilities of the ruler and the compass had brought mosaic artists to the point of drawing each figure with the use of ruler and compass. The conscious use of geometric subjects as patterns shows that basic mathematical concepts were applied in the arts.

The construction of a whole mosaic with the use of a primary pattern can be considered as an induction application. Algorithm generating activities executed with the repetition of a particular operational procedure are parallel to the primary pattern repeating activities. The differential and integral concepts of modern mathematics are based on the idea that a change occurring in one part induces changes on the whole and there is a ratio between the changes occurring in the part and on the whole (Boyer 1949: 187). There are specific examples of the interaction between the part and the whole in the design and construction phases of the mosaic works.

In this paper, it is shown that the creation of complex patterns was achieved with the use of a primary pattern within a specific order. Presently, the idea that there is an order within anything chaotic is being studied within the framework of the chaos theory (Wiggins 2003). This theory supports the idea that there are rooted mental processes, whose traces can be found in the arts. These applied, tested, grasped and accustomed processes are subject to man's creative thinking in the field of mathematics, in a way that mathematics knowledge reflects on the arts.

3. Conclusion

In the ancient times, in the absence of a common symbolic language in mathematics, the transfer of mathematical knowledge, the discussion of problems and the finding of solution methods was achieved with the help of symbols (Heath 1921: 26-64). However, on account of the fact that grasping mathematical concepts and processing mathematical knowledge only with the use of symbols can be quite difficult, designed geometric figures were also been used. As in the case of the Pythagorean theorem representation by a right triangle and the representation of the square duplication problem by a square duo, mathematical knowledge was fictionalized and encoded by figures.

In order to shed light on the history of mathematical studies and the knowledge level of ancient communities, it is important to study art works with direct and indirect reflection of mathematical knowledge. This study draws attention to the connections that can be revealed by basic geometric figures reflected into art while at the same time sheds light on conscious applications of mathematical knowledge in the mosaic arts. It provides specific examples which may be used in studies questioning the relation between art history and mathematics.

The conclusion being drawn is that famous figures and mathematical methods have been used as a tool in search of beauty, harmony and aesthetics, just like in the case of famous mythological figures and their stories. Mathematical knowledge has been hidden in colour transitions, in optical illusions and symmetries. The order has been hidden in chaos and the artistic result has come to the foreground. All mosaics with a geometric pattern can be analyzed and common aspects of these mosaics can be determined. Furthermore, mosaics conservation studies may be supported by this geometric model.

This study has revealed the existence of a deformation concept with the use of a simple theoretical relation between the geometric plan and its application. Evidently, the deformation concept has quite significant results and can be a quite useful tool for mosaic researches. Measurement works and a formulated approach provides a set of digital data and becomes a meaningful factor. Because of the fact that geometric studies can also provide significant knowledge as far as measurement units used in the ancient era is concerned. Furthermore, it is also estimated that the knowledge obtained by geometric studies on mosaic floors can be really useful for conservation works.

Bibliography - Kaynaklar

Aybek et al. 2009	S. Aybek – A. E. Meriç – A. K. Öz, A Mother Goddess City in Ionia: Metropolis, İstanbul.
Beryozin 1994	V. N. Beryozin, "The Good Old Pythagorean Theorem", Quantum, Jan./Feb. 94, 25-28.
Boyer 1949	C. B. Boyer, The History of the Calculus and its Conceptual Development, New York.
Heath 1921	T. L. Heath, A History of Greek Mathematics, Vol I, From Thales to Euclid, Oxford.
Lang-Auinger 1996	C. Lang-Auinger, Das Hanghaus 1 in Ephesos. Der Baubefund, Forschungen in Ephesos, VIII/3, Vienna.
Lietzmann 1966	W. Lietzmann, Der Pythagoreische Lehrsatz mit einem Ausblick auf das Fermatsche Problem, Stuttgart.
Meriç 1999	R. Meriç, "Metropolis 1997 Yılı Kazı Raporu", 20. KST II, 335-352.
Meriç 2004	R. Meriç, Metropolis: City of the Mother Goddess, İstanbul.
Nelson 1993	R. B. Nelson, Proofs Without Words, Washington.
Öz 2012a	A. K. Öz, "The Research and Conservation Study of the Mosaics of the Roman Bath at Metropolis", JMR 5, 145-162.
Öz 2012b	A. K. Öz, "The Mosaics of Metropolis in Ionia", M. Şahin, (ed.), The Proceedings of V. International Mosaic Corpus of Türkiye, 11th International Colloquium on Ancient Mosaics, İstanbul, 701-708.
Parrish 2007	D. Parrish, "The General State of Mosaic Research in Turkey in 2005", M. Şahin (ed.), The Proceedings of III. International Symposium of the Mosaic of Turkey, Bursa, 17-26.
Radt 1988	W. Radt, Pergamon: Geschichte und Bauten einer antiken Metropole, Kö ln.
Veljan 1993	D. Veljan, The 2500 Year-old Pythagorean Theorem, Washington.
Vitr.	Vitriuvius, The Ten Books on Architecture, (Translated by M. H. Morgan), London, Oxford University Press, 1914.
Wiggins 2003	S. Wiggins, Introduction to Applied Dynamical Systems and Chaos, New York.