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DYNAMIC RESPONSE ANALYSIS OF A 3-STORY SHEAR FRAME SUBJECTED TO HARMONIC LOADING: AN ANALYTICAL APPROACH

Ahmet TUKEN *

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Abstract: The multi-story buildings are likely to be subjected to various types of dynamic loadings. A sinusoidal external force with a certain frequency applied to a system is considered as harmonic excitation. Since it is faced very commonly and also covers the concept of resonance, the response of a system to harmonic excitation is an important topic. In the present study, a three-story shear frame subjected to a harmonic force at the top floor is studied and the equations for the floor displacements as functions of time have been obtained. Then normalized response amplitudes under the applied harmonic loading is plotted against the frequency ratio ω/ω_1. These frequency-response curves show three resonance conditions at $\omega=\omega_1$, $\omega=\omega_2$ and $\omega=\omega_3$; at these exciting frequencies the steady-state response is unbounded. At other exciting frequencies, the vibration is finite and could be calculated from the derived equations. The structure experiences resonance at some frequency if the structure is excited with harmonic loading over a range of frequencies. When the frequency of the excitation is equal to the natural frequency of the structure, resonance occurs. The structure experiences its largest response at the resonant frequency as compared to any other frequency of loading. If the loading frequency is close to, not exactly equal to, the natural frequency of the system, a phenomenon known as "beating" may occur. In this kind of vibration, the amplitude builds up and then diminishes in a regular pattern.

Keywords: Harmonic loading, Shear frame, Frequency-response curves, Normalized response, MDOF system.

Harmonik Yüklemeye Maruz Kalan 3-Katlı Kayma Çerçevesinin Dinamik Tepki Analizi: Analitik Bir Yaklaşım

Öz: Çok katlı binaların çeşitli tiplerde dinamik yüklemelere maruz kalması muhtemeldir. Bir sisteme uygulanan belli bir frekansta sinüzoidal bir dış kuvvet harmonik uyarma olarak kabul edilir. Çok sık karşılaşıldığı ve aynı zamanda rezonans kavramını da kapsadığı için, bir sistemin harmonik uyarmalara tepkisi önemli bir konudur. Bu çalışmada, en üst katta harmonik bir kuvvete maruz kalan üç katlı bir kayma çerçevesi çalışılmış ve zamanın fonksiyonu olarak kat yer değiştirmeleri için denklemler elde edilmiştir. Daha sonra uygulanan harmonik yükleme altındaki normalleştirilmiş tepki büyüklükleri, frekans oranı ω/ω_1'e karşı çizilmiştir. Bu frekans-tepki eğrileri, $\omega=\omega_1$, $\omega=\omega_2$ ve $\omega=\omega_3$ değerlerinde üç rezonans durumu gösterir; bu uyarıcı frekanslarda kararlı-durum tepkisi sınırsızdır. Diğer uyarıcı frekanslarda, titreşim sonludur ve türetilmiş denklemlerden hesaplanabilir. Yapı bir dizi frekans üzerinden harmonik yükleme ile uyarıldığında, bazı frekanslarda rezonans yaşar. Uyarıcı frekans yapının doğal frekansına eşit olduğunda rezonans oluşur. Rezonans frekansında, yapı diğer herhangi bir yükleme frekansına kıyasla en büyük tepkisini tecrübe eder. Yükleme frekansı, sistemin doğal frekansına çok yakın ama tam olarak eşit değilse, "atma" olarak bilinen bir olgu oluşabilir. Bu tür bir titreşimde, genlik yükselir ve daha sonra düzenli bir şekilde azalır.

* Department of Civil Engineering, King Saud University, Riyadh 11421, Saudi Arabia
Correspondence Author: Ahmet TUKEN (atuken@ksu.edu.sa)

Anahtar Kelimeler: Harmonik yükleme, Kayma çerçevesi, Frekans-tepki eğrisi, Normalleştirilmiş tepki, Çok serbestlik dereceli sistem (ÇSDS)

1. INTRODUCTION

Shear frames are used in all the major reinforced concrete buildings and structures in all over the world. These frames are subjected to various statics and dynamic loads. The most common static load is the gravity load identified as dead and live loads. Among the dynamic loads, we have high probable wind and wave loads and less probable earthquake, impulsive and harmonic loads. A sinusoidal external force with a certain frequency applied to a system is considered as harmonic excitation. Since it is faced very commonly and also covers the concept of resonance, the response of a system to harmonic excitation is an important topic. When the external excitation frequency and the natural frequency of the system are the same, resonance occurs and it leads to large displacements that can cause a system to exceed its elastic range and structurally fail at the end.

A substantial research exists on response of shear frames and buildings subjected to wind, wave, earthquake and impulsive loads. (Tuken, 2018) analytically formulated the response of a 3-story shear frame subjected to impulsive loading. He applied a rectangular pulse force at the top floor and obtained the equations for the floor displacements as functions of time. Then the floor displacements under the applied rectangular pulse force is plotted against time. He observed that the displacement values at the top floor where the impulsive load is applied is maximum and also observed that the peak value of the response at the top floor is obtained at the end of the rectangular pulse force duration. (Elhelloty, 2017) carried out the transient and modal analysis to investigate the effect of lateral loads resisting systems on response of buildings subjected to dynamic loads. Three and five stories steel frame buildings with and without three lateral loads resisting systems that are laminated composite plate shear walls, steel bracings and steel plate shear walls subjected to dynamic loads are examined for total displacement and equivalent stresses with respect to time history graphs, natural frequencies and mode shapes. Comparative study is performed to investigate the effect of lateral loads resisting systems on the performance of buildings subjected to dynamic loads using the finite element software ANSYS. (Patel ve diğ., 2017) reviewed the work carried out in past few years on blast effects on structures. A blast explosion inside or surrounding the structure can cause severe damage to the structural and non-structural members. The structure can be made blast resistant but not a blast proof in reality and also it is not an economical option. The main goal of this study is to elucidate the improvement of building security against the effects of explosives in structural design process, the design techniques that should be carried out and the design theories for blast resistant building. The paper also includes introduction and detail explanation on blast wave phenomenon as well as review on various research carried out in the past on blast load and their effect on the structures.

The above review shows that there is a substantial research available on the response of shear frames and buildings subjected to wind, wave, earthquake and impulsive loads, but the researches on shear frames and buildings subjected to harmonic loads are limited. (Baig ve diğ., 2014) studied a fifteen storey structure bare frame by adopting the harmonic response technique on ANSYS platform and assessed the displacements of the structure at various floor levels by employing mode superposition method. Peak displacement is then visualized by means of the frequency v/s displacement graph, which is obtained from mode superposition of reduced modes at forcing frequencies. (Abbas ve diğ., 2017) investigated the dynamic response of gas turbine-generator foundation of Al-Mansurya power plant station in Iraq under the vertical harmonic excitation by considering the layered-soil with linear elastic model and using the finite element modelling with and without considering the soil-structure interaction effect. In order to find the natural frequencies and the corresponding mode shapes, free vibration analysis is also performed in addition to force vibration analysis. The response was evaluated both at the foundation and the soil during the operational conditions using ABAQUS software. It was observed that machine operating frequency should be far different from the natural frequency of the turbine foundation in order to avoid resonance. The response of the system with and without considering the soil-structure interaction effect were also compared for the gas turbine foundation under the vertical harmonic excitation. It was concluded that when analyzing such sensitive structure, the soil-structure interaction must be considered because of its significant effect on the overall response.

The above computational researches on harmonic loads are primarily based on advanced numerical (e.g. finite element or boundary element) software. Simple analytical methods were used in very limited studies for obtaining the response of shear frames or buildings subjected to harmonic loads. In the present study, the effect of harmonic load on a 3-story shear frame was studied using a simple analytical approach. The shear frame consists of beams and columns rigidly connected at the ends. The columns which are fixed at the base primarily provide the lateral stiffness. For dynamic analysis, the entire shear frame was modelled as Multi Degree of Freedom (MDOF) system having rigid beams and lumped masses of each story placed at the middle of the beams.

2. ANALYTICAL FORMULATION

In this section, analytical formulation of a 3-story shear frame is presented for obtaining its response against harmonic loading. The dimensions, floor masses and storey stiffnesses of this frame are shown in Fig. 1. This shear frame is subjected to a harmonic force $p(t)=p_0.\sin\omega t$ at the top floor. The equations for the floor displacements as functions of time is derived and the normalized response amplitudes are plotted against the frequency ratio ω/ω_1.

2.1. Mass Matrix

Since the beams are rigid in flexure and axial deformation is neglected in columns, three DOFs associated with each story represent the properties of this three-story shear frame. The corresponding story masses are:

$$m_1 = m; \quad m_2 = m; \quad m_3 = \frac{m}{2} \tag{1}$$

Using the lumped mass concept, mass matrix is obtained easily as follows:

$$\mathbf{m} = m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \tag{2}$$

Here, m = 45 t = 45,000 kg (as shown in Fig. 1)

2.2. Stiffness Matrix

The stiffness coefficients k_{i1}, k_{i2} and k_{i3} were obtained by applying unit displacement at each degree of freedom respectively as shown in Fig. 2. Keeping all the stiffness coefficients in the form of a matrix, we obtain the stiffness matrix as below:

$$\mathbf{k} = k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \tag{3}$$

where,

$k = 2\left(\frac{12EI}{h^3}\right) = \frac{24EI}{h^3} = 57{,}000\ \text{kN/m}$; E = modulus of elasticity of concrete; I = moment of inertia of each column and h = story height.

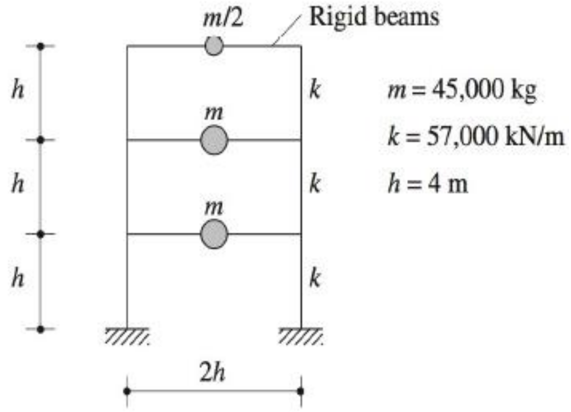

Figure 1:
Schematic view of the 3-story shear frame

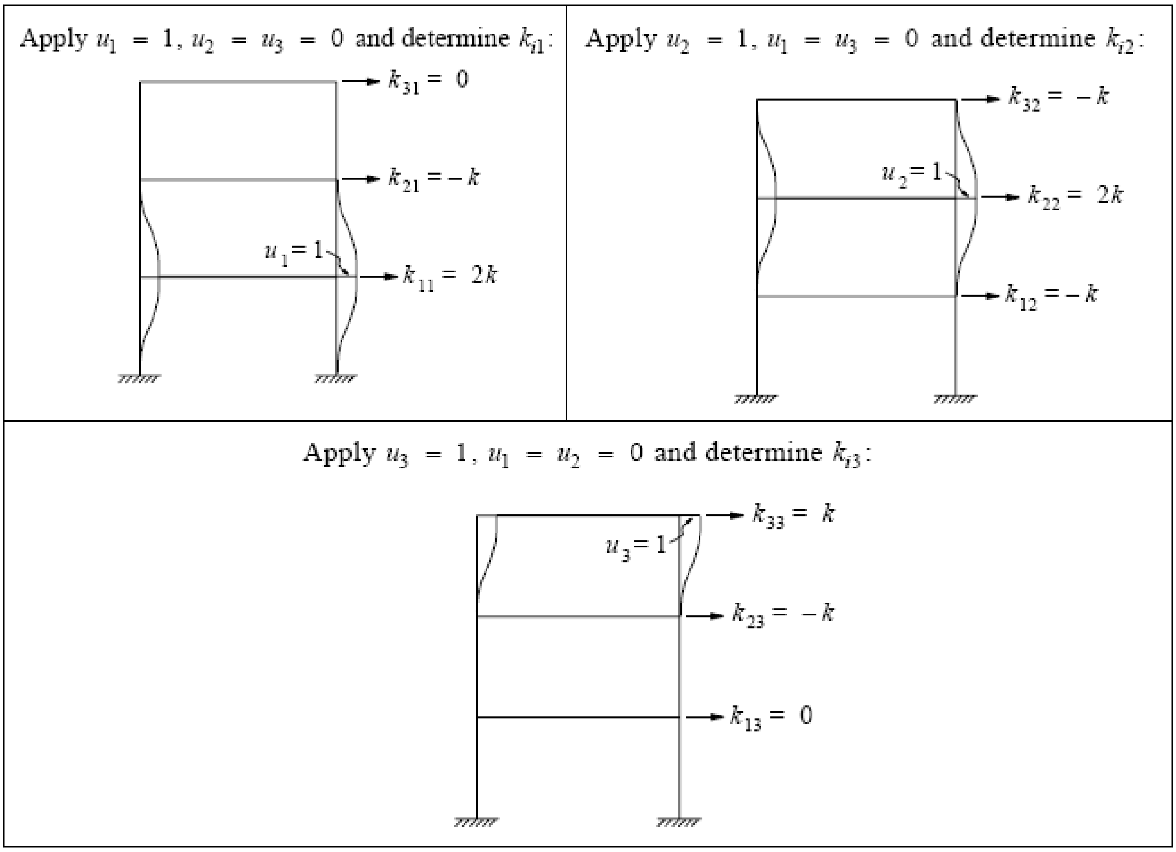

Figure 2:
Determination of stiffness coefficients

2.3. Force Matrix

In the case of harmonic loading, the forcing function is given by P(t)= P_0.Sin(ωt) where P_0 is the amplitude and ω is the circular frequency of the harmonic load. In the present study, the harmonic load is applied at the top floor. Therefore, force matrix is expressed as

$$\mathbf{P(t)} = \begin{Bmatrix} 0 \\ 0 \\ p_0 \end{Bmatrix} \sin \omega t \tag{4}$$

Having derived the mass, stiffness and force matrices, one can write the equation of motion as given in the following section.

2.4. Equation of Motion

Using the above derived matrices, the governing equation of motion for the undamped shear frame under harmonic loading, applied at the top floor, can be written as follows:

$$m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}\begin{Bmatrix} \ddot{u_1} \\ \ddot{u_2} \\ \ddot{u_3} \end{Bmatrix} + k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ p_0 \end{Bmatrix} \sin \omega t \tag{5}$$

The above equation of motion will be solved to obtain the desired response as given in the following section.

3. RESPONSE ANALYSIS

The response of the undamped shear frame structure subjected to harmonic load is obtained following the procedure presented in Dynamics of Structures by (Chopra, 2014). In the following subsections, natural frequencies and mode shapes of the structure will be determined and the response of the structure will be obtained.

3.1. Natural Frequencies and Mode Shapes

The natural frequencies and mode shapes can be determined from

$$\mathbf{k} - \omega^2\mathbf{m} = \frac{24EI}{h^3}\begin{bmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-0.5\lambda \end{bmatrix} \tag{6}$$

where $\lambda = \frac{mh^3}{24EI}\omega^2$

det $[\mathbf{k} - \omega^2\mathbf{m}]$ = 0 gives the frequency equation as follows

$$\lambda^3 - 6\lambda^2 + 9\lambda - 2 = 0 \tag{7}$$

The solution of this 3rd degree frequency equation (Eqn. 7) gives

$$\lambda_1 = 2 - \sqrt{3} = 0.2679;\ \lambda_2 = 2;\ \lambda_3 = 2 + \sqrt{3} = 3.7321 \tag{8}$$

The corresponding natural frequencies are as below:

$$\omega_1 = 2.5359\sqrt{\frac{EI}{mh^3}};\ \omega_2 = 6.9282\sqrt{\frac{EI}{mh^3}};\ \omega_3 = 9.4641\sqrt{\frac{EI}{mh^3}} \tag{9}$$

And the mode shapes are obtained as follows:

$$\emptyset_1 = \begin{Bmatrix} 0.5 \\ 0.866 \\ 1 \end{Bmatrix} \qquad \emptyset_2 = \begin{Bmatrix} -1 \\ 0 \\ 1 \end{Bmatrix} \qquad \emptyset_3 = \begin{Bmatrix} 0.5 \\ -0.866 \\ 1 \end{Bmatrix} \tag{10}$$

3.2. Determination of Response

Often, a steady state is approached asymptotically. If the convolution system is stable, the response to a sinusoidal input is asymptotically sinusoidal, with the same frequency as the input, and therefore the steady-state response is assumed as

$$\begin{Bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{Bmatrix} = \begin{Bmatrix} u_{10} \\ u_{20} \\ u_{30} \end{Bmatrix} \sin \omega t \tag{11}$$

Where

$$\begin{Bmatrix} u_{10} \\ u_{20} \\ u_{30} \end{Bmatrix} = \left[\mathbf{k} - \omega^2 \mathbf{m}\right]^{-1} \begin{Bmatrix} 0 \\ 0 \\ p_0 \end{Bmatrix} = \frac{1}{\det[\mathbf{k}-\omega^2\mathbf{m}]} \operatorname{adj}\left[\mathbf{k} - \omega^2 \mathbf{m}\right] \begin{Bmatrix} 0 \\ 0 \\ p_0 \end{Bmatrix} \tag{12}$$

In which,

$$\begin{aligned} \det\left[\mathbf{k} - \omega^2 \mathbf{m}\right] &= m_1.m_2.m_3.(\omega_1^2 - \omega^2).(\omega_2^2 - \omega^2).(\omega_3^2 - \omega^2) \\ &= \frac{1}{2} m^3 (1- \omega^2/\omega_1^2).(1- \omega^2/\omega_2^2).(1- \omega^2/\omega_3^2)\, \omega_1^2\, \omega_2^2\, \omega_3^2 \\ &= k^3 (1- \omega^2/\omega_1^2).(1- \omega^2/\omega_2^2).(1- \omega^2/\omega_3^2) \end{aligned} \tag{13}$$

And

$$\operatorname{adj}\left[\mathbf{k} - \omega^2 \mathbf{m}\right] \begin{Bmatrix} 0 \\ 0 \\ p_0 \end{Bmatrix} = k^2 p_0 \begin{Bmatrix} 1 \\ 2(1 - \frac{\omega^2}{\omega_2^2}) \\ 4\left(1 - \frac{\omega^2}{\omega_2^2}\right) - 1 \end{Bmatrix} \tag{14}$$

Therefore,

$$\begin{Bmatrix} u_{10} \\ u_{20} \\ u_{30} \end{Bmatrix} = \frac{p_0}{k} \frac{1}{(1 - \frac{\omega^2}{\omega_1^2})(1 - \frac{\omega^2}{\omega_2^2})(1 - \frac{\omega^2}{\omega_3^2})} \begin{Bmatrix} 1 \\ 2(1 - \frac{\omega^2}{\omega_2^2}) \\ 4\left(1 - \frac{\omega^2}{\omega_2^2}\right) - 1 \end{Bmatrix} \tag{15}$$

Finally, the floor displacements can be obtained by the following equations

$$\begin{Bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{Bmatrix} = \frac{p_0}{k} \frac{1}{(1-\frac{\omega^2}{\omega_1^2})(1-\frac{\omega^2}{\omega_2^2})(1-\frac{\omega^2}{\omega_3^2})} \begin{Bmatrix} 1 \\ 2(1-\frac{\omega^2}{\omega_2^2}) \\ 4\left(1-\frac{\omega^2}{\omega_2^2}\right)-1 \end{Bmatrix} \sin\omega t \tag{16}$$

On the other hand, using the following definition for C_n

$$C_n = \frac{1}{(1-\frac{\omega^2}{\omega_n^2})} \tag{17}$$

Normalized response amplitudes can be expressed as follows:

$$\frac{u_{10}}{p_0/k} = C_1 C_2 C_3$$

$$\frac{u_{20}}{p_0/k} = 2C_1 C_3 \tag{18}$$

$$\frac{u_{30}}{p_0/k} = C_1 C_2 C_3 (\frac{4}{C_2^2} - 1)$$

4. RESULTS AND DISCUSSION

The normalized response amplitudes obtained in eqn.(18) were plotted against the frequency ratio ω/ω_1 as shown in Fig. 3 where ω_1 is the fundamental natural frequency of the system. For ω/ω_n associated with modes 2 and 3, the deformation response factor is closer to 1.0 implying that the response contributions of modes 2 and 3 may be determined by static analysis; that is, the response due to only the first mode need to be determined by dynamic analysis. These frequency-response curves show three resonance conditions at $\omega=\omega_1$, $\omega=\omega_2$ and $\omega=\omega_3$; at these exciting frequencies the steady-state response is unbounded (i.e. the response of the system will be very large). This condition should be avoided to prevent failure of the system. At other exciting frequencies, the vibration is finite and could be calculated from the above three expresions given in eqn.(16). If the loading frequency is close to, not exactly equal to, the natural frequency of the system, a phenomenon known as "beating" may occur. In this kind of vibration, the amplitude builds up and then diminishes in a regular pattern. There is an exciting frequency where the vibration of the first mass, along which the exciting force is applied, is reduced to zero. This is the entire basis of the dynamic vibration absorber or tuned mass damper.

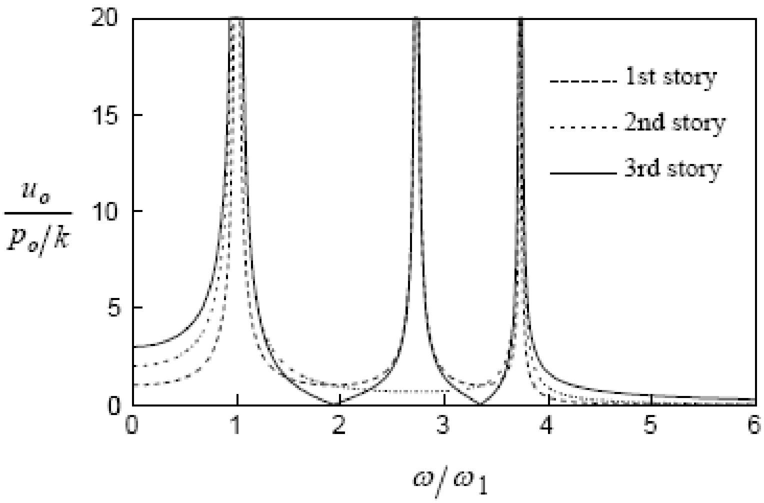

Figure 3:
Normalized response amplitudes against the frequency ratio ω/ω_1

5. CONCLUSIONS

Followings are the important outcomes of the present study:

- The structure experiences resonance at some frequency if the structure is excited with harmonic loading over a range of frequencies. When the frequency of the excitation is equal to the natural frequency of the structure, resonance occurs. The structure experiences its largest response at the resonant frequency as compared to any other frequency of loading.
- For ω/ω_n associated with modes 2 and 3, the deformation response factor is closer to 1.0 implying that the response contributions of modes 2 and 3 may be determined by static analysis; that is, the response due to only the first mode need to be determined by dynamic analysis.
- Frequency-response curves show three resonance conditions at $\omega=\omega_1$, $\omega=\omega_2$ and $\omega=\omega_3$; at these exciting frequencies the steady-state response is unbounded. At other exciting frequencies, the vibration is finite and could be calculated from the derived equations.
- Also, it was observed that the amplitude of the motion changes with the excitation frequency. The amplitude of the vibrations will become very large as the excitation frequency get closer to the natural frequency.
  - If the loading frequency is close to, not exactly equal to, the natural frequency of the system, a phenomenon known as "beating" may occur. In this kind of vibration, the amplitude builds up and then diminishes in a regular pattern.
  - There is an exciting frequency where the vibration of the first mass, through which the exciting force is applied, is reduced to zero.

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