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## FINITE ELEMENT STRESS AND MODAL ANALYSIS OF BIMETAL SPUR GEARS

*Tufan Gürkan Yılmaz\** 

*Fatih Karpaz\** 

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Received: 22.07.2019; revised: 24.10.2019; accepted: 18.11.2019

**Abstract:** Involute spur gears subject to damage on their fillet and contact areas because of local stresses. There is no significant stress except these regions. Based on this fact, different materials could be used for high stress and low stress regions provided that suitable fusion method is available. This kind of design change results in weight reduction, and it ensures to reduce CO<sub>2</sub> emissions for vehicles. In this study, natural frequencies and root stress of bimetal spur gears were analyzed. Results were compared with full steel spur gear. The steel was used for high stress region. Aluminum and magnesium alloys were used as low stress zone.

**Key Words:** Bimetal spur gear, natural frequencies, finite element analysis

### Çift Metalli Dişlilerin Sonlu Elemanlar Gerilme ve Modal Analizi

**Öz:** Evolvent dişli çarklar diş dibi ve temas bölgelerindeki gerilmelerden ötürü hasara maruz kalırlar. Bunun haricinde kalan bölgelerde kayda değer bir gerilme bulunmaz. Bu temel üzerine bir dişli çarkın yüksek ve düşük gerilme bölgeleri uygun bir birleştirme metodu ile farklı malzemelerden imal edilebilir. Bu tarz bir tasarım değişikliği ağırlıkta azalma sağlayarak CO<sub>2</sub> gazı emisyonlarını düşürür. Bu çalışmada çift metalli dişli çarkların doğal frekansları ve diş dibi gerilmeleri analiz edilmiştir. Sonuçlar geleneksel çelik malzemeden imal edilmiş dişli çarka ait sonuçlar ile karşılaştırılmıştır. Düşük gerilme bölgesinde alüminyum ve magnezyum alaşımları kullanılmıştır.

**Anahtar Kelimeler:** Çift metalli dişliler, doğal frekanslar, sonlu elemanlar analizi

## 1. INTRODUCTION

Weight reduction is an essential goal to decrease CO<sub>2</sub> emission and fuel costs. Material substitution is one the most effective ways of it (Yılmaz et al., 2017). This alteration is feasible for components which subject to low stress and deformation. However, for highly stressed components such as gears, this change could be partially (Politis et al., 2014). Stresses occur locally in spur gears; bending, shear, and compressive stress in root regions and Hertz stress in contact region. Except these regions, there is scarcely any stress. Based on this point, the low density materials could be used for low stress regions of spur gear to achieve reducing weight. Among these materials aluminum alloys come into prominence with its mechanical and physical properties. Aluminum alloys have adequate strength to density ratio. In addition, components could be manufactured with aluminum by traditional methods (casting, forging, machining etc.). Politis et al. investigated hot forged bimetal spur gears with aluminum alloy as low stress region

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\* Bursa Uludağ University, Faculty of Engineering, Department of Mechanical Engineering, Gorukle, 16059, Bursa.

\* Bursa Uludağ University, Faculty of Engineering, Department of Mechanical Engineering, Gorukle, 16059, Bursa.

Correspondence Author: Tufan Gürkan Yılmaz (tufanyilmaz@uludag.edu.tr)

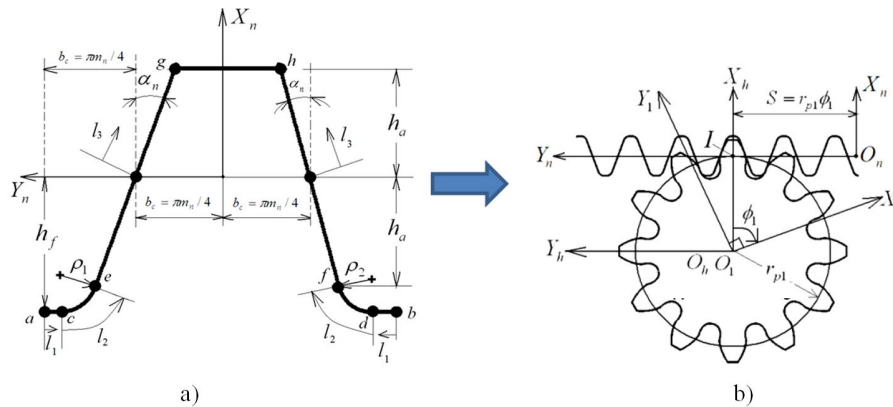
material. They stated that increasing steel ring thickness leads to decrease root stress, but this situation leads to increase weight. As to contact stress, it remains nearly stable with changing ring thickness (Politis et. al., 2018). Wu et al. proved to manufacturability of steel/aluminum spur gear design. The effect of hot forging parameters on the steel/aluminum gear was investigated numerically (Wu et al., 2017). The density and elasticity modulus of Magnesium alloys is lower than aluminum alloys. However, it is generally weaker than aluminum alloys. In addition the manufacturability of magnesium is more laborious than aluminum as high temperatures are required for forming (Politis et. al., 2018).

In this study, the effect of core material on root stress and natural frequency of bimetal spur gears were examined numerically. 3D finite element models were realized in CATIA. Finite element analyses were conducted for bimetal spur gears. Root Stress and natural frequencies were obtained and compared.

## 2. METHOD

### 2.1 Design of Bimetal Gears

The mathematical equations of spur gear were derived based on Litvin’s approach (Litvin and Fuentes, 2004). This approach uses the geometry of generating rack cutter, coordinate transformation, differential geometry and theory of gearing. In Figure 1, the geometry of rack cutter and relationship between cutter and gear blank is presented.



**Figure 1:**

a) The geometry of rack cutter, b) Relationship between rack and gear

The equations for involute and trochoid curves that constitute gear tooth were presented in Eq. 1 to Eq. 6.

*Trochoid curve*

$$x_1^{\text{trochoid}} = (-h_f + \rho_1 - \rho_1 \cos l_2) \cos \phi_1 - (b_c + h_f \tan \alpha_n - \rho_1 \tan \alpha_n + \rho_1 \sec \alpha_n - \rho_1 \sin(l_2)) \sin \phi_1 + r_{p1} \cos \phi_1 + r_{p1} \phi_1 \sin \phi_1 \quad (1)$$

$$y_1^{\text{trochoid}} = (-h_f + \rho_1 - \rho_1 \cos l_2) \sin \phi_1 + (b_c + h_f \tan \alpha_n - \rho_1 \tan \alpha_n + \rho_1 \sec \alpha_n - \rho_1 \sin(l_2)) \cos \phi_1 + r_{p1} \sin \phi_1 - r_{p1} \phi_1 \cos \phi_1 \quad (2)$$

$$\varnothing_1 = \frac{(b_c + h_f \tan \alpha_n - \rho_1 \tan \alpha_n + \rho_1 \sec \alpha_n - \rho_1 \sin(l_2) - ((-h_f + \rho_1 - \rho_1 \cos l_2) \tan(l_2))}{r_{p1}} \quad (3)$$

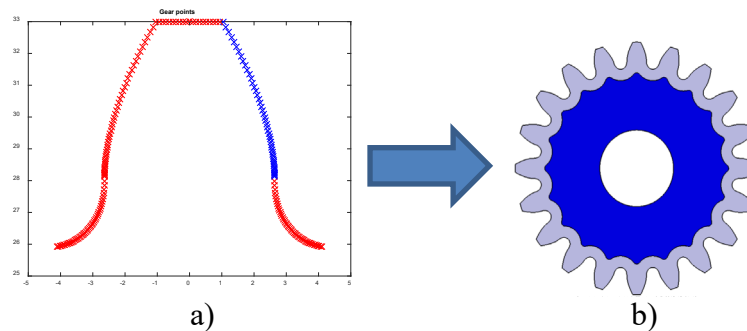
*Involute curve*

$$x_1^{\text{involute}} = l_3 \cos \alpha_n \cos \varnothing_1 - (b_c - l_3 \sin \alpha_n) \sin \varnothing_1 + r_{p1} \cos \varnothing_1 + r_0 \varnothing_1 \sin \varnothing_1 \quad (4)$$

$$y_1^{\text{involute}} = l_3 \cos \alpha_n \sin \varnothing_1 + (b_c - l_3 \sin \alpha_n) \cos \varnothing_1 + r_{p1} \sin \varnothing_1 - r_{p1} \varnothing_1 \cos \varnothing_1 \quad (5)$$

$$\varnothing_1 = \frac{-l_c \cos \alpha_n + b_c - l_c \sin \alpha_n}{r_{p1} \tan \alpha_n} \quad (6)$$

Where;  $h_f$  is dedendum,  $h_a$  is addendum,  $b_c$  is half of the tooth thickness at pitch circle,  $\rho_1$  and  $\rho_2$  is tip radius on the right and left sides,  $\alpha_n$  is pressure angle,  $r_{p1}$  is radius of pitch circle,  $l_2$  and  $l_3$  are design variables of trochoid and involute curve respectively,  $\varnothing_1$  is roll angle. A MATLAB code was prepared to obtain coordinate points of gear tooth. These points were sent to CATIA for 3D design of spur gear with two materials. In Figure 2, coordinates of points of gear tooth and 3D design view are illustrated.



**Figure 2:**  
*a) Points of gear tooth, b) 3D spur gear design with two materials*

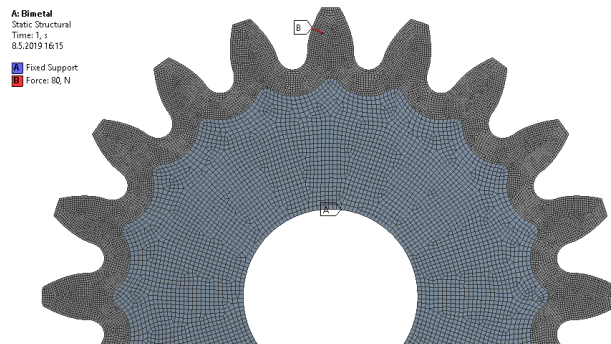
## 2.2 Finite Element Analyses

To understand the core material effect on stress and natural frequencies, several finite element analyses were conducted using ANSYS Workbench software. In Table 1, the gear parameters used in analyses are presented.

**Table 1. Parameters for case study**

Parameters	Case I	Case II	Case III
<b>m (mm)</b>	3	3	3
<b><math>Z_p-Z_g</math></b>	20-20	20-20	20-20
<b><math>\alpha</math></b>	20°	20°	20°
<b><math>h_a</math> (mm)</b>	1xm	1xm	1xm
<b><math>h_f</math> (mm)</b>	1.25xm	1.25xm	1.25xm
<b><math>p_r</math> (mm)</b>	Fully rounded	Fully rounded	Fully rounded
<b>x</b>	0	0	0
<b>b (mm)</b>	1	1	1
<b>Ring rim thickness (mm)</b>	Solid rim	3	3
<b>Shaft hole diameter (mm)</b>	20	20	20
<b>Ring material</b>	Steel	Steel	Steel
<b>Core material</b>	Steel	Aluminum alloy	Magnesium alloy

The full solid model was used for finite element both stress and natural frequencies. Normal load (80 N) was implemented at highest point of single tooth contact (HPSTC). No displacement was allowed in shaft hole region. 0.25 and 0.5 hexahedral mesh sizes were used for ring and core region, respectively. Boundary conditions and mesh distribution were presented in Figure 3.



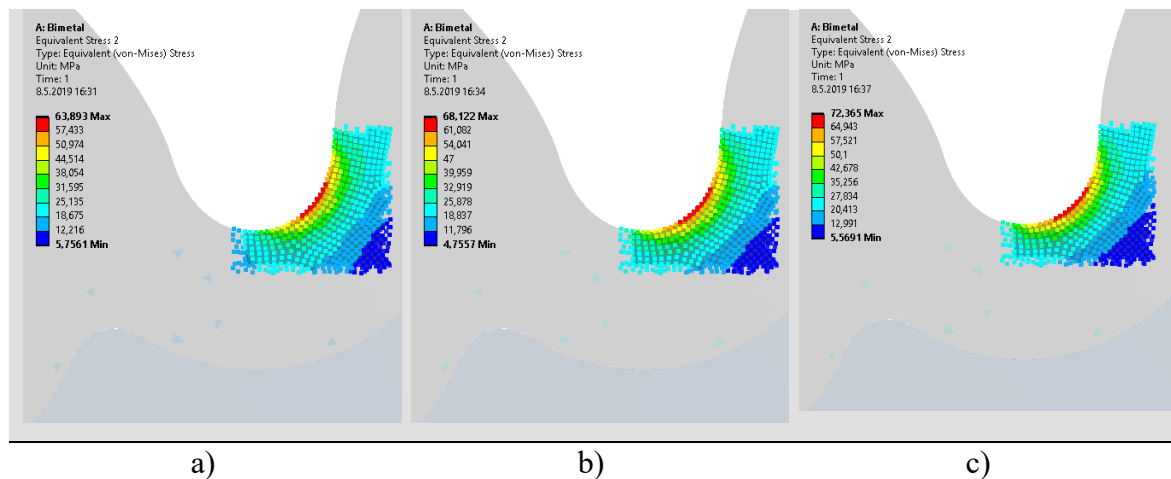
**Figure 3:**

*Mesh and boundary conditions of stress analysis*

For modal analysis, 0.5 mm hexahedral element size was used for the meshing process. It was assumed free-free boundary condition that allows displacement and rotation from all directions. Young modulus was taken for aluminum alloy 72 GPa while 45 GPa for magnesium alloy. Poisson ratios were taken as 0.33 and 0.35 for aluminum and magnesium alloy, respectively. For steel, 200 GPa young modulus and 0.3 poisson ratio values were assumed.

### 3. RESULTS AND DISCUSSION

In this section, the root stresses were obtained for different core materials. As the tensile side of the gear tooth is important for tooth breakage, only stress values on this side are illustrated in Figure 4.



**Figure 4:**  
 Root stress results; a) Case I, b) Case II, c) Case III

According to stress results when the aluminum alloy is used for the core region, the root stress nearly increases by 7% with regard to full steel. For magnesium, the increase ratio is more than aluminum. It is approximately 13%. Actually, it is an expected result as Aluminum alloy is stiffer material than magnesium alloy. The stress and weight situation of cases were also presented in Table 2.

**Table 2. Stress and weights of cases**

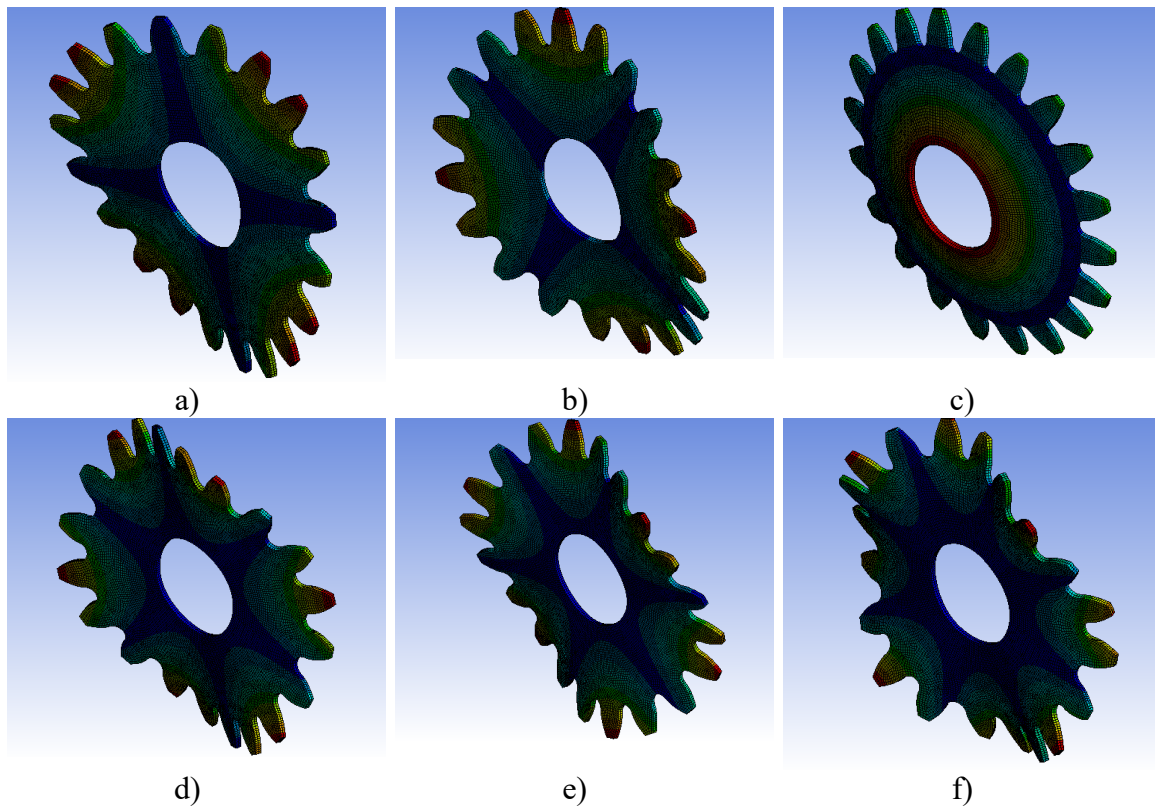
Case	Root Stress (MPa)	Weight (kg)
I	63.898	0.019442
II	68.122	0.011804
III	72.365	0.010452

The effect of core material on natural frequencies was investigated by the finite element method. The first six modes are related to translational and rotational rigid body movements that these modes are not taken into account. The natural frequencies of modes from 7 to 12 are presented for different core materials in Table 3.

**Table 3. Natural frequencies of different modes**

Modes	Case I (Hz)	Case II (Hz)	Case III (Hz)
7	1171.5	905.4	814.2
8	1178.4	911.2	818.3
9	2064.3	1852.6	1811.8
10	2775.3	2208.6	2031.8
11	2780.1	2212.4	2035.4
12	4367.6	3772.9	3539.9

The density of magnesium alloy and aluminum alloy are 1.8 g/cm<sup>3</sup> and 2.77 g/cm<sup>3</sup>, respectively. Steel’s density is 7.85 g/cm<sup>3</sup>. For this reason, the weight of gear with magnesium core is the lowest among them. In addition, the young modulus of magnesium is also nearly one to four of steel and half of aluminum density. Based on these facts, the results presented in Table 3 are expected. The natural frequencies were reduced with decreasing weight and material stiffness. Mode shapes were also given in Figure 5.



**Figure 5:**  
*Mode shapes; a) 7, b) 8, c) 9, d) 10, e) 11, f) 12*

#### 4. CONCLUSION

In this study, the effect of core material on bimetallic spur gear's root stresses and natural frequencies were studied. Finite element analyses were conducted for this aim. The 3D design was constituted in CATIA and analyses were conducted in ANSYS Workbench. According to results, the weight of gear could be decreased by using bimetal design. However the stress value increases nearly 7% for aluminum alloy and 13% for magnesium alloy with respect to full steel gear. Natural frequencies also decrease with using bimetal gear design. The seventh natural frequency of spur gear with aluminum alloy has 30% lower value than steel one. This ratio is 43% for spur gear with magnesium core. It is concluded that bimetal spur gear design should be arranged in view of stress and natural frequencies. Ring rim thickness could be increased to decrease stress values.

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