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COOPERATION, EIGENVALUES, REPELLOR AND ATTRACTOR

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SUMMARY

In this paper we derive some general conditions for a polygon of orientable hypersurfaces to be a repellor (respectively attractor) using modern geometric methods.

Key Words: Manifold, Hypersurface, Cooperation, Orientable. 1980 Subject Classification: 53AOB.

ÖZET

Ortaklık, Eigen Değerler, Repellor ve Attractor

Bu makalede, geometrik yöntemler kullanılarak, yön koruyan hiper-yüzeylerin bir poligonu için Repellor ve Attractor olma koşullarını verdik.

INTRODUCTION

Although the matereal used in this scientific paper under the title "Co-operation, Eigenvalues, Repellor and Attractor" is geometric the result obtained cast light on many subjects such as medical sciences, population genetics, pre-biotic development, differential equations, applied mathematics, physics and modern differential geometry.

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1. COOPERATION, EIGENVALUES, REPELLOR AND ATTRACTOR

Let us consider a C1-flow on a 2-dimensional orientable manifold M which exhibits a finite number of cyclically connected 2-hypersurfaces. Of course the flow is then not structurally stable, but such situations often occur in concrete dynamical systems (defined on compact subsets of IR^n where the boundry is invariant).

Let L be this connected invariant set consisting of n 2-hypersurfaces F_1, F_2, ..., F_i, ..., F_{n-1}, F_n and n connecting orbits. Since the manifold is orientable, a certain neighbourhood U of L may be embedded diffeomorphically in the space IR^2. Let V be the component of U/L which lies "inside" the polygon L. An orbit starting in V (Close to L) which is not closed may have L as α-or as ω-limit.

One can choose "coordinates" X_{ij}: M $\rightarrow$ IR such that $x_{ij} > 0$ in V, $X_{ij} = 0$ along the orbit connecting F_{i-1} with F_i and finally $X_{i1} \times X_{i2}$ is a diffeomorphism of a neighbourhood of F_i on to a neighbourhood of the origin in IR^2. Then consider the vector field near the 2-hypersurface F_i: Along the orbit $X_{i1} = X_{i2} = 0$ we have

$$\frac{d}{dt} X_{i1} (X (t)) \sim k_{i1} X_{i1} (X(t)) \text{ near } X_{i1} = 0, \text{ ...., and along}$$

$X_{i1} = X_{i2} = 0$ we have

$$\frac{d}{dt} X_{i2} \ (X(t) \ \sim k_{i2} X_{i2} (X(t)) \text{ near } X_{i2} = 0, \text{ where } k_{ij} \neq 0, i = 1, ...,$$

n; j = 1, 2 are the Eigenvalues at the 2-pypersurface point.

A more detailed discussion of these and other notions may be found in [1], [2], [3], [4], [5], [6] and [7].

1.1. Proposition. Consider the function

$$P = X^{p_{11}}{}_{11} \ X^{p_{12}}{}_{12} \ ..... X^{p_{i1}}{}_{i1} \ X^{p_{i2}}{}_{i2} \ ..... X^{p_{n1}}{}_{n1} \ X^{p_{n2}}{}_{n2}$$

where $p_{ij} > 0$ will be specified later which is positive on V and equal to 0 on L. Then, we have

$$\psi (x) = \frac{\dot{P}}{P} = \sum_{i, j=1}^{n, 2} pij \frac{\dot{X}_{ij}}{X_{ij}} .$$

PROOF. Differentiating the function P with respect to t we have

$$\dot{P} = P_{11}\dot{X}_{11}X^{-1}{}_{11}P + ... + P_{i1}\dot{X}_{i1}X^{-1}{}_{i1}P + ... + P_{n2}\dot{X}_{n2}X^{-1}{}_{n2}P.$$

Hence the proof is completed.

1.2. Proposition. The function

$$\psi(x) = \frac{\dot{P}}{P} = \sum_{i,j=1}^{n,2} pij \frac{\dot{X}_{ij}}{X_{ij}}.$$

reduces at the i-th corner to

$$\psi(F_i) = p_i \lambda_i + p_{i+1} \mu_i$$

where $\lambda_i > 0$ and $\mu_i < 0$ are the Eigenvalues at the saddle point F_i.

PROOF. Let F_i be the 2-hypersurface such that

$$\varphi \mid_{(x, y)} = (x, y, xy).$$

and X_1, X_2 be the orthhonormal basis vectors of TF_i (0) at the origin. Let us define the unit normal vector field of F_i by

$$\xi = \frac{(-y, -x, 1)}{\sqrt{1 + x^2 + y^2}}$$

If we derivate the function $\varphi(x, y)$, we have:

$$\varphi_x = (1, 0, y)$$

$$\varphi_y = (0, 1, x).$$

It is clear that $< \zeta, \varphi_x > = <\zeta, \varphi_y > = 0$ and $|| \zeta || = 1$. Hence ζ is really an unit normal vector field of the 2-hypersurface F_i. At the origin, we have

$$\varphi_x \mid_0 = (1, 0, 0)$$

$$\varphi_y \mid_0 = (0, 1, 0)$$

$$\zeta \mid_0 = (0, 0, 1).$$

Then, $\zeta_x|_0 = x_1$, $\zeta_y|_0 = x_2$. Now we can find the vector $S \mid X_1$,

$$S\,|_{X_1} = -\nabla_{X_1}\zeta = -\zeta'(0 + tX_1)\,|_{t=0} = -\{\frac{(0, -t, 1)}{\sqrt{1 + t^2}}\}'\,|_{t=0}$$

$$S|_{X_1} = \frac{(0, 1, 0)}{\sqrt{1 + t^2}}\,|_{t=0} + \{(0, t, -1)\,(-\frac{1}{2})\,(1 + t^2)^{-3/2}\,2t\}\,|_{t=0}$$

$$S|_{X_1} = (0, 1, 0) \quad \text{or} \quad S|_{X_1} = X_2\,.$$

In the same way, we may have $S|_{X_2} = X_1$.

Let $\alpha = aX_1 + bX_2, -\alpha = -aX_1 - bX_2, \beta = a\ X_1 - bX_2, -\beta = -aX_1 + bX_2$ be the tangent vectors of F_i at the origin. Then we find

$$k|_{\alpha} = < S\alpha, \alpha > = \frac{1}{a^2 + b^2} < aX_2 + bX_1, aX_1 + bX_2 > = \frac{2ab}{a^2 + b^2}$$

$$k|_{-\alpha} = \quad < S\,(-\alpha), -\alpha > = \frac{2ab}{a^2 + b^2}$$

$$k|_{\beta} = < S\beta, \beta > = \frac{1}{a^2 + b^2} < aX_2 - bX_1, aX_1 - bX_2 > = -\frac{2ab}{a^2 + b^2}$$

$$k|_{-\beta} = < S\,(-\beta), -\beta > = -\frac{2ab}{a^2 + b^2}$$

where S denotes the shape operator of the 2-hypersurface F_i. On the other hand, it is clear that

$$k|_{X_1} = < SX_1, X_1 > = < X_1, X_2 > = 0$$

$$k|_{X_2} = < SX_2, X_2 > = < X_1, X_2 > = 0\,.$$

From the above results we understand that there exists the Eigenvalues $\lambda_i > 0$, $\mu_i < 0$.

The function ψ (x) continous since $\dot{x}_j = 0$ for $x_j = 0$ and the vector field $\dot{x}_j$ is C^1.

In view of the above remarks, we may write

$$\psi(F_i) = p_{i1}k_{i1} + p_{i2}k_{i2}.$$

If we take, $p_{i1} = p_i, p_{i2} = p_{i+1}, k_{i1} = \lambda_i > 0, k_{i2} = \mu_i < 0$ we may say that the proof is completed.

1.3. Proposition. Consider n 2-hypersurfaces. Let $F_1, F_2, ..., F_i, ..., F_n$ be the 2-saddles. Let L be the above polygon, $\lambda_i > 0$ and $\mu_i < 0$ the Eigenvalues of the 2-saddles and let $\upsilon = \prod_{i=1}^{n} (-\frac{\lambda_i}{\mu_i})$. Then L is a repellor if and only if $1 < \upsilon$.

PROOF. At the i-th corner, we may write

$$\psi(F_i) = p_i \lambda_i + p_{i+1} \mu_i$$

which is positive if and only if

$$\frac{p_{i+1}}{p_i} < -\frac{\lambda_i}{\mu_i}$$

that is

$$\psi(F_i) = p_i \lambda_i + p_{i+1} \mu_i > 0 \Leftrightarrow \frac{p_{i+1}}{p_i} < -\frac{\lambda_i}{\mu_i}$$

$$\Leftrightarrow \frac{p_{i+1}}{p_i} = -\frac{\lambda_i}{\mu_i} . \upsilon^{-1/n} < -\frac{\lambda_i}{\mu_i}$$

$$\Leftrightarrow \prod \frac{p_{i+1}}{p_i} = \prod_{i=1}^{n} (-\frac{\lambda_i}{\mu_i}) \underbrace{\upsilon^{-1/n} ...... \upsilon^{-1/n}}_{\text{n-times}} < \prod_{i=1}^{n} (-\frac{\lambda_i}{\mu_i})$$

$$\Leftrightarrow \upsilon\upsilon^{-1} < \upsilon$$

$$\Leftrightarrow 1 < \upsilon.$$

Then the proof is completed.

1.4. Proposition. Consider n 2-hypersurfaces. Let $F_1, F_2, ..., F_i, ..., F_n$ be the 2-saddles. Let L be the above polygon, $\lambda_i > 0$ and

$\mu_i < 0$ the Eigenvalues of the 2-saddles and let $\upsilon = \prod_{i=1}^{n} (-\frac{\lambda_i}{\mu_i})$. Then L is a repellor if and only if $\upsilon < 1$.

PROOF. Again, at the i-th corner, we have

$$\psi(F_i) = p_i \lambda_i + p_{i+1} \mu_i$$

which is negative if and only if

$$\frac{p_{i+1}}{p_i} > -\frac{\lambda_i}{\mu_i}.$$

From this, we may find

$$\psi(F_i) = p_i \lambda_i + p_{i+1} \mu_i < 0 \Leftrightarrow \frac{p_{i+1}}{p_i} > -\frac{\lambda_i}{\mu_i}$$

$$\Leftrightarrow \frac{p_{i+1}}{p_i} = -\frac{\lambda_i}{\mu_i} \cdot \upsilon^{-1/n} > -\frac{\lambda_i}{\mu_i}$$

$$\Leftrightarrow \prod_{i=1}^{n} \frac{p_{i+1}}{p_i} = \prod_{i=1}^{n} (-\frac{\lambda_i}{\mu_i}) \underbrace{\upsilon^{-1/n} \ldots\ldots \upsilon^{-1/n}}_{\text{n-times}} > \prod_{i=1}^{n} (-\frac{\lambda_i}{\mu_i})$$

$$\Leftrightarrow \upsilon\upsilon^{-1} > \upsilon$$

$$\Leftrightarrow \upsilon < 1.$$

Hence the proof is completed.

In these proofs we have used various preliminary results which may be found in [6], [7], [8], [9] and [10].

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