Buckling of Composite Plates With U-shaped Cutouts

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ABSTRACT: The influence of cutout shape upon the buckling stability of multilayered rectangular epoxy plates reinforced by glass fiber, with different orientation angles is investigated. U-shaped cutouts are made on the long sides symmetrically. The investigated plates are simply supported on the loaded edges (i.e. short sides) and free on the unloaded edges. The plates without cutouts are examined theoretically to confirm experimental and Finite Element (FE) results. The FE and experimental results are found out for different U-shaped cutouts sizes. U-notch shape effects are examined depending on notch depth and notch root radius. According to results, the effect of notch depth is stronger than that of the notch root radius on buckling loads of plates. But, in some cases, although plates containing U-notch, no reduction is obtained in buckling loads.

Obtained results with experimental, theoretical, and FE are in good agreement with each other.

KEY WORDS: U-shaped notch, buckling, composite laminates.

INTRODUCTION

FLAT RECTANGULAR PANELS with cutouts are found in several aircraft structural components such as wing ribs and spars. Due to practical requirements, cutouts are often necessary in these panels to form ports for access, inspection, electrical lines, and fuel lines or to reduce the overall weight of the aircraft. Because of their lightweight and tailorability, composite materials are becoming increasingly more popular for the manufacturing of these panels.

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Intuition may suggest that introducing a large hole into a compression-loaded plate can cause a reduction in the buckling load of the plate. Previous studies of the buckling behavior of composite plates with circular cutouts, such as the study by Yasui and Tsukamura [1] have shown that introducing a hole into a plate does not always reduce the buckling load and, in some instances, may increase its buckling load. Nemeth [2] predicted the buckling behavior of laminated angle-ply plates with holes under compression stress and displacement loads using an approximate analysis. Lin and Kuo [3] also presented a finite element analysis for the buckling of symmetrically or antisymmetrically laminated composite plates with circular holes under inplane static loadings. Britt [4] investigated the shear and compression loading of rectangular composite laminates with centrally located elliptical cutouts. Srivatsa and Murty [5] used a new finite element to study stability of laminates with cutouts. Noor et al. [6] presented a different study which focuses on understanding the detailed buckling response characteristics of multilayered composite panels with cutouts, subjected to combined mechanical and thermal loads. Lee and Hyer [7] presented a paper summarizing a study focused on understanding the failure mechanisms present in a plate with a centrally located circular hole loaded in-plane into the postbuckling range of deflections. Bailey and Wood [8] examined the effect of different sized and different shaped cutouts on the buckling and postbuckling responses of graphite-epoxy panels with cutouts loaded in uniaxial compression. Square and circular shaped cutouts were used. However, to the authors' knowledge, no known work presently exists on the buckling of anisotropic plates with U-shaped cutouts.

This paper examines the buckling behavior of compression-loaded rectangular unidirectional angle-ply plates with U-shaped cutouts on long sides. Using the Finite Element analysis, results of a parametric study are presented that illustrate the influence of reinforcing angle and U-notch dimensions. ANSYS, commercial general purpose 'finite element analysis system' with laminated element was utilized for the solution. The FE results are compared with experimental and analytical values.

THEORETICAL BUCKLING ANALYSIS OF UNNOTCHED ORTHOTROPIC COMPOSITE PLATES

Equation of orthotropic plate under in-plane distributed pressure load, (p) (see Figure 1) was obtained as [9],

$$D_1 \frac{\partial^4 w}{\partial x^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4} + p \frac{\partial^2 w}{\partial x^2} = 0 \tag{1}$$

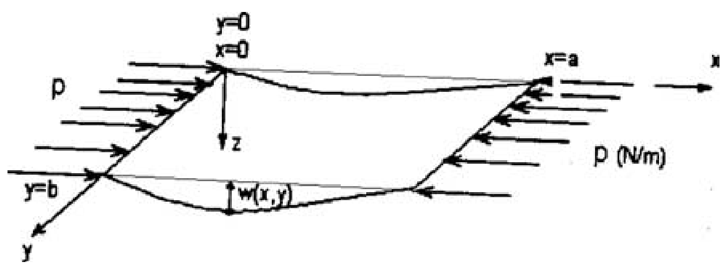


Figure 1. *Buckling of orthotropic plate subjected to p distributed loads in x-direction at x–y plane.*

where $w(x, y)$ is deflection of plate middle plane. The elastic constants for orthotropic materials can be given as

$$D_1 = \frac{E_1 h^3}{12(1 - \upsilon_{12}\upsilon_{21})} \quad D_2 = \frac{E_2 h^3}{12(1 - \upsilon_{12}\upsilon_{21})} \tag{2}$$

$$D_k = \frac{G_{12} h^3}{12} \quad D_3 = D_1 \upsilon_{21} + 2D_k$$

Distributed bending moment and distributed shear forces in the x-section of plate, which normal to x direction, can be expressed as

$$M_x = -D_1\left(\frac{\partial^2 w}{\partial x^2} + \upsilon_{21}\frac{\partial^2 w}{\partial y^2}\right) \tag{3}$$

$$N_x = -\frac{\partial}{\partial x}\left(D_1\frac{\partial^2 w}{\partial x^2} + D_3\frac{\partial^2 w}{\partial y^2}\right) \tag{4}$$

Moreover, in the y-section of plate, which normal to y direction, distributed bending moment and distributed shear forces are

$$M_y = -D_2\left(\frac{\partial^2 w}{\partial y^2} + \upsilon_{12}\frac{\partial^2 w}{\partial x^2}\right) \tag{5}$$

$$N_y = -\frac{\partial}{\partial x}\left(D_1\frac{\partial^2 w}{\partial x^2} + D_3\frac{\partial^2 w}{\partial y^2}\right) \tag{6}$$

These force and moments are shown in Figure 2.

If the parallel edges with y-axis are assumed simply supported, the boundary condition can be given as

$$\begin{array}{ll} \text{(a) } w(0, y) = 0 & \text{(c) } w(a, y) = 0 \\ \text{(b) } M_x(0, y) = 0 & \text{(d) } M_x(a, y) = 0 \end{array} \tag{7}$$

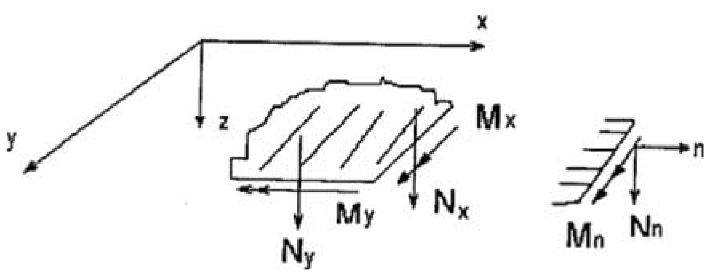


Figure 2. *Forces and moments in plate.*

Solution of Equation (1) can be suggested as

$$w(x, y) = \varphi(x)f(y) \tag{8}$$

Under given boundary conditions, the function $\varphi(x)$ can be written as

$$\varphi_m(x) = \sin\frac{m\pi x}{a} \tag{9}$$

Finally, suitable $f(y)$ function was obtained as

$$f(y) = A\cosh k_1 y + B\sinh k_1 y + C\cos k_2 y + D\sin k_2 y \tag{10}$$

Since parallel edges to x-axis were free, the boundary conditions can be given as

$$\begin{aligned} &\text{(a) } M_y(x, 0) = 0 \quad \text{(c) } M_y(x, b) = 0 \\ &\text{(b) } N_y(x, 0) = 0 \quad \text{(d) } N_y(x, b) = 0 \end{aligned} \tag{11}$$

Under boundary conditions, the unknown coefficients in the Equation (10) were found. Finally, the critical buckling load for orthotropic plates at first buckling mode is obtained in the following compact form:

$$p_{cr} = D_1\left(\frac{\pi}{a}\right)^2 \tag{12}$$

where P_{cr} is the critical buckling load. D_1 is the orthotropic plate stiffness, which is given in Equation (2).

PANEL DETAILS

The composite ply consists of LY5082 epoxy resin matrix (Ciba Geigy) and E-Glass fiber. E-glass fiber volume fractions in the composites were fixed at 30%. The composite plies were laid up to form three-ply laminates having $[\theta]_3$ stacking sequences. Fiber orientations in the ply were taken at 0, 30, 45, 60, and 90° ccw directions from x axes. Laminates were cured in an oven using the manufacturer's recommended procedures (at 80°C throughout 8 h). After curing, the laminates were machined into test specimens.

The mechanical properties of the composite layer were measured by using strain gages, INSTRON 4301 universal tensile testing machine, and a macro extensometer by using testing methods as given in literature [10]. Obtained composite material properties are presented in Table 1 for $[0°]_3$ reinforced unidirectional composites.

All specimens were 200 mm long × 100 mm wide and the loaded edges were machined flat and parallel to permit uniform compressive loading (See Figure 3). The panels have a nominal thickness of 2.25 mm. U-shaped cutouts were machined into the long sides of the composite panels using high-speed milling. U-shape were produced at different dimensions by taking $r = 1$ mm and $a = 0$, 1, 2 cm.

Table 1. Lamina properties of the glass fiber/epoxy composite plates.

E_{11} (GPa)	E_{22} (GPa)	G_{12} (GPa)	ν_{12}	ν_{21}
25.04	4.49	1.619	0.333	0.06

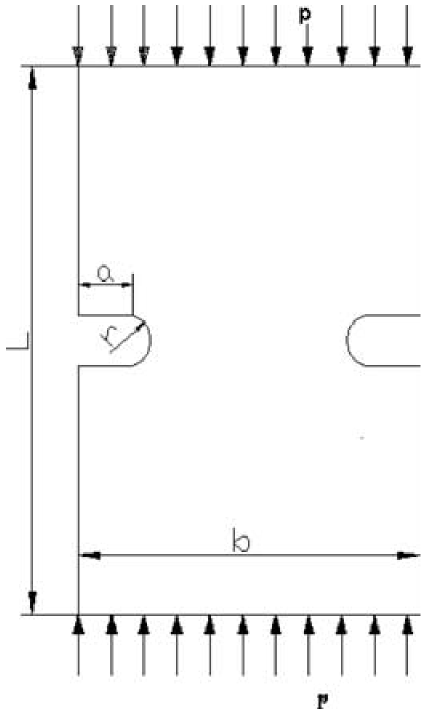


Figure 3. *Plate geometry with parametric dimensions.*

BUCKLING EXPERIMENT

In order to verify the finite element and theoretical results a limited experimental program has been carried out. The experimental program is ongoing and to date the panels tested include an unperforated ($r = 0$, $a = 0$), and three type notched panels with $r = 1$ $a = 0$, $r = 1$ $a = 1$, and $r = 1$ $a = 2$ cm .The panels consists three plies which have 0, 30, 45, 60 and 90° fiber orientation angles. The experimental investigations were carried out in a purpose built panel buckling test rig as shown in Figure 4. The test rig utilized an INSTRON 4301 universal uniaxial tension–compression testing machine. The test rig has been designed to apply compression load with simply supported edges. The unloaded edges of the panel were free. The panels were loaded slowly at 1 mm/min rate and critical buckling load were obtained. To determine the initial buckling load two methods are commonly used, firstly the point of inflection in the plot of load against in-plane displacement and secondly the point of reversal in the membrane strain. Buskell et al. [11] showed good agreement between the two methods. In these buckling tests, the point of inflection in the load–deflection plot was used.

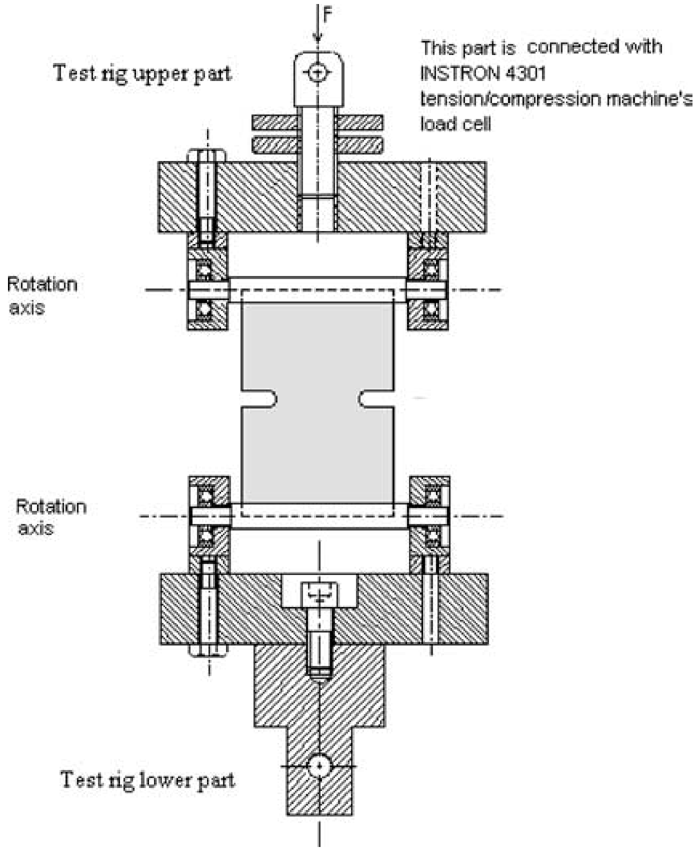


Figure 4. *Top and bottom supports.*

FINITE ELEMENT STUDY

Eigenvalue buckling assumes that the structure is perfect; that is to say that it is straight, uniform, and free from eccentric loading and that a point of bifurcation exists. The general form of the eigenvalue problem for buckling can be expressed as follows [12].

$$([K] + \lambda_i[S])(\Psi)_i = 0 \tag{13}$$

A numerical buckling study of rectangular panels with U-shaped cutouts was performed using ANSYS 5.5. Notch dimensions were taken in the FE analysis as $r_i(a_j)$ for $r_i = 0$, 1, 2 and $a_j = 0$, 1, 2 cm. Totally nine FE models were analyzed. The panel was modeled with eight noded SHELL91, multilayered shell elements, having six degrees of freedom per node, as shown in Figure 5. The investigated plates were simply supported on the loaded edges (i.e. short sides) and free on the unloaded edges. The prebuckled stress distribution was then evaluated under 1 N/m uniform load and saved for the eigenvalue buckling solution. The eigenvalue buckling load was determined. This procedure was carried out for all plate geometries.

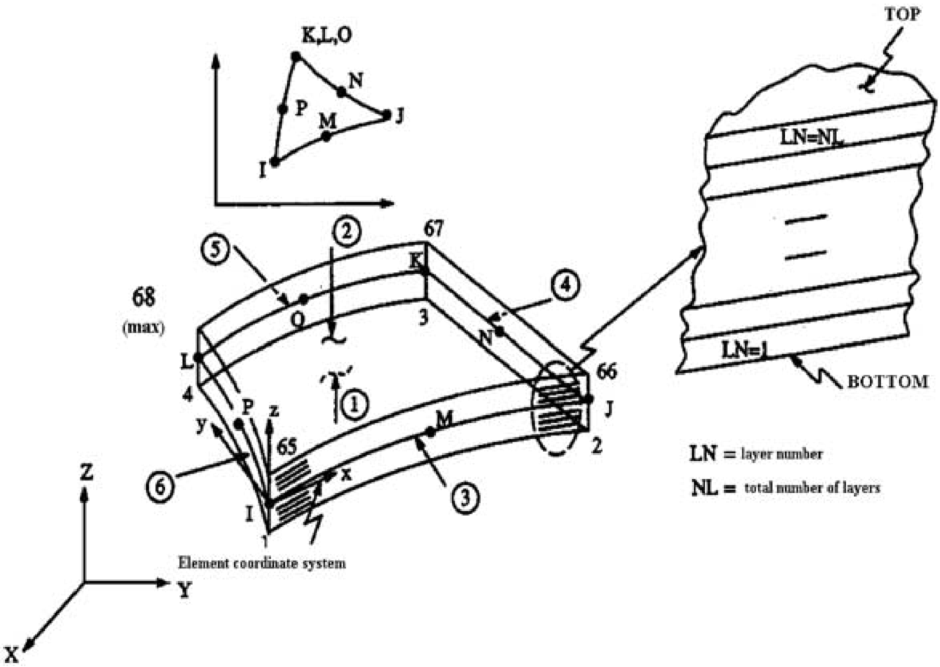


Figure 5. *Element specification.*

RESULTS AND DISCUSSION

Comparison between the FE, experimental, and theoretical results for in-plane responses of unperforated plates are shown in Figure 6. The buckling load is plotted as a function of fiber orientation angle, which is varied from 0 to 90°. Experimental, FE, and Analytical result for the unnotched composite plates show good correlation. Therefore, the FE solutions can be used for finding cutout size effect.

Figure 7(a)–(c) shows influence of fiber angle θ on critical buckling loads with above-mentioned boundary conditions for various U-shaped cutout root radii. The curve, which is shown in Figure 7(a)–(c) with 0–90° orientation angles, indicates a decrease in the critical buckling load for all U-shaped cutout sizes. Because, the bending rigidity of plates are decreasing with varying fiber orientation angles from 0° to 90°. Figure 8(a)–(c) also show effect of fiber angle θ on critical buckling loads for various U-shaped notch depths. Figures 9(a)–(c) and 10(a)–(c) shows variation of critical buckling load by U-shaped notch root radius and U-shaped notch depth respectively. The increase in loss of bending stiffness due to increase in cutout size yields a reduction in buckling resistance. However, for zero U-shaped cutout depth or root radius (See Figures 9(a) and 10(a)), the critical buckling loads of plate increased for values of θ bigger than 45°. The critical buckling load ratios for Perforated to Unperforated plates are shown in Table 2. The minimum value of P_{cr}(Perf.)/P_{cr}(Unperf.) ratio (0.21) are obtained for $[0°]_3$ reinforced composite plates with maximum cutout size ($a=2$, $r=2$). It means that the buckling capacity of plates is decreased about 79% due to the U-shaped notch. In some cases, the critical buckling loads of perforated panels are obtained higher than that of the unperforated composite panels. The P_{cr}(Perf.)/P_{cr}(Unperf.) ratios are obtained generally bigger or around 1 for composite

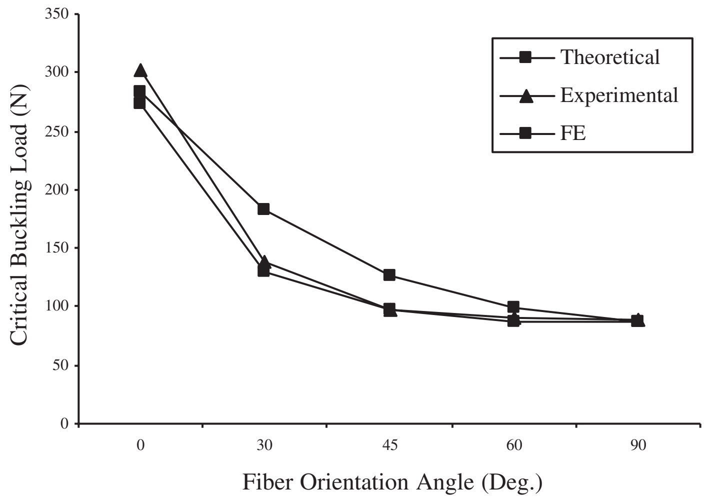


Figure 6. *Comparison of FE, experimental, and theoretical critical buckling loads of unperforated composite plates.*

plates having fiber orientation angles of $\theta \geq 45°$. The maximum ratio (1.46) is observed for $[90°]_3$ fiber reinforced plates perforated with $r = 1$ cm, $a = 0$ cm cutout. Correlation between U-shaped perforation root radius and depths are shown in Figure 11(a)–(e) for values of θ. Similar trends are shown for the composites with increasing notch root radius and notch depth for 0–30° and 45–90° ply orientation angles. The results in Figure 11(a)–(e) show a monotonic reduction in buckling load with increasing notch depth (a) or notch root radius (r) for the all plates except for having cutout sizes of $r = 0$ cm or $a = 0$ cm dimensions.

Comparison between FE analysis and experimental results for very limited U-shaped cutouts are given in Figure 12(a)–(c). It is observed that from Figure 12(a)–(c), the critical buckling loads show good agreement for especially composite plates having fiber-reinforced angles of ply bigger than 45°. Overall effect of cutout shape, as seen in Figure 13 has great influence on the buckling response of the panels. The effect of notch depth is bigger than that of the notch root radius on buckling loads of plates. This means that, the critical buckling ratio of the semi circular shaped cutouts is bigger than rectangular shaped crack (width equal 0 cm) which is perforated on lateral edge of the plates.

CONCLUSIONS

In this study, the stability analysis of composite plates has been carried out by using FE, physical testing, and theoretical methods. The investigated plates are simply supported on the loaded edges (i.e. short sides) and free on the unloaded edges. Effect of semi circular, U-shaped cutouts and rectangular cracks, which are perforated on lateral edges center of the plates, have been investigated by using parametric cutout dimensions. According to the authors' knowledge, this work is the first study about buckling of composite plates having lateral U-shaped cutouts. Moreover, the theoretical buckling analysis of unperforated

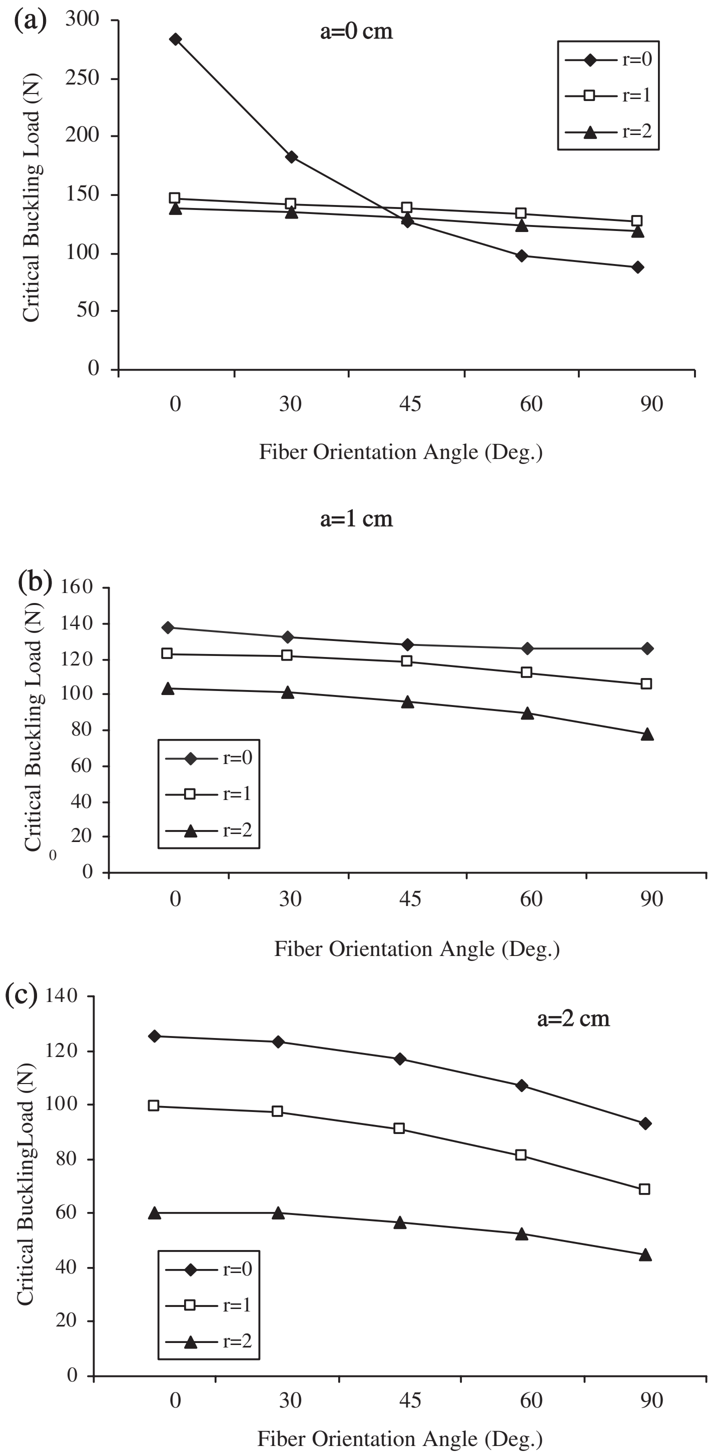


Figure 7. *Variation of critical buckling load via fiber orientation angle for U-shaped cutout depth of (a) a = 0 cm; (b) a = 1 cm; (c) a = 2 cm.*

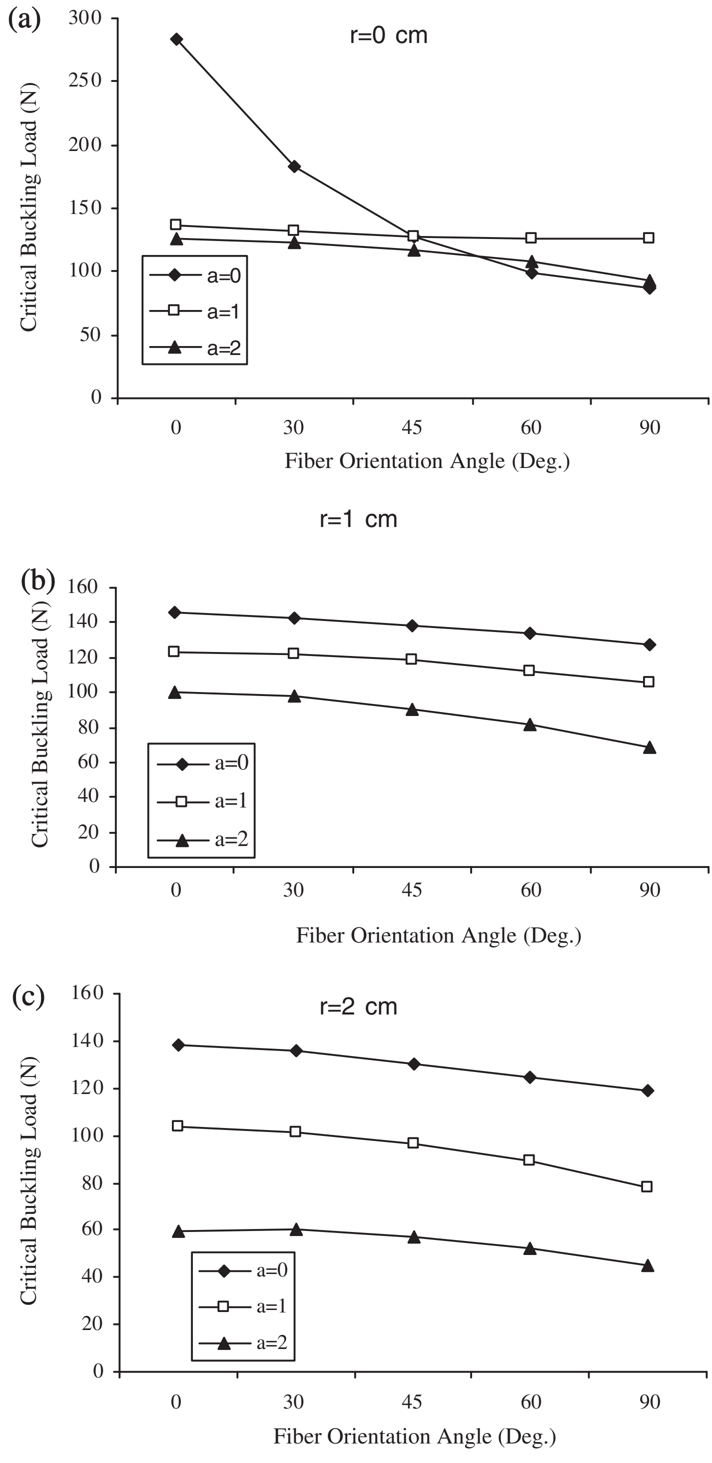


Figure 8. *The critical buckling load variation according to fiber orientation angle for U-shaped cutout root radius for (a) $r = 0$ cm; (b) $r = 1$ cm; (c) $r = 2$ cm.*

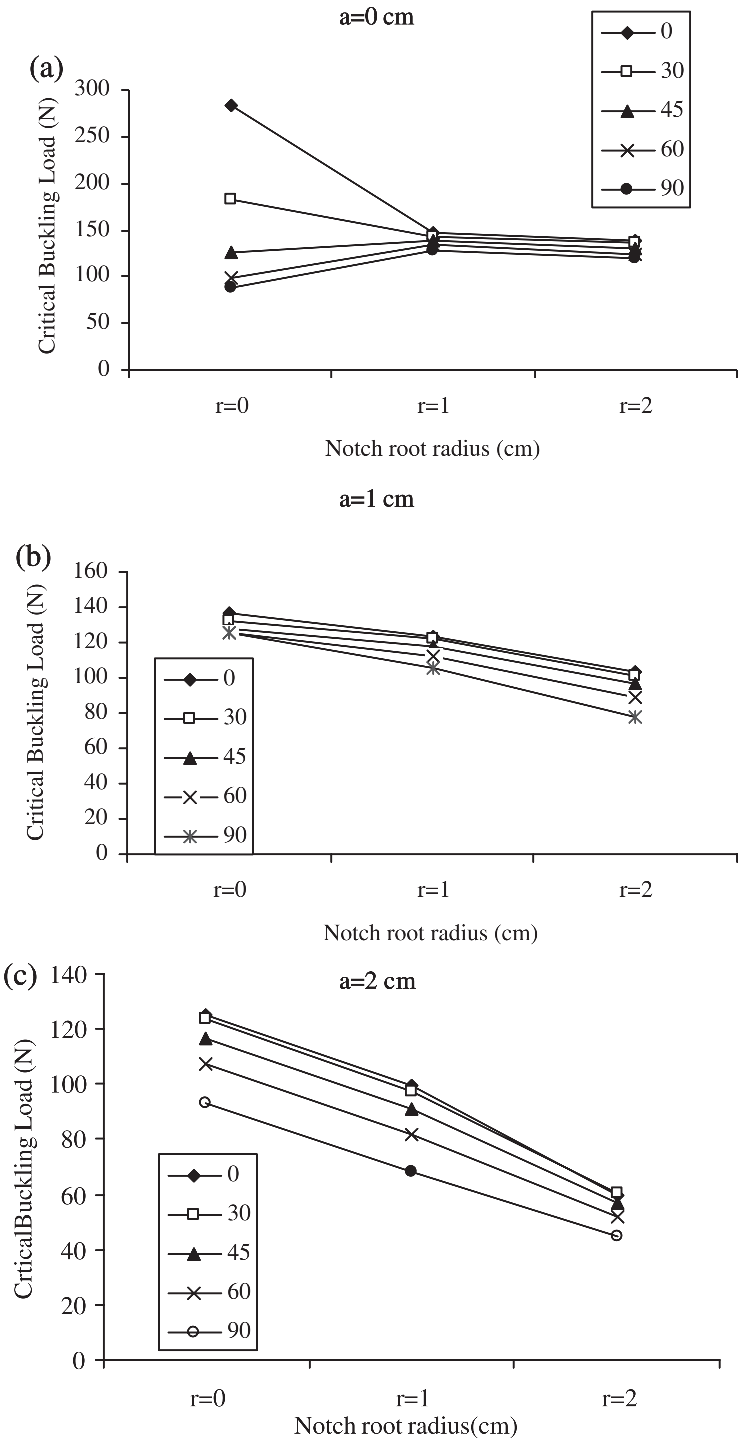


Figure 9. *The critical buckling load variation according to U-shaped cutout root radius for (a) a = 0 cm; (b) a = 1 cm; (c) a = 2 cm.*

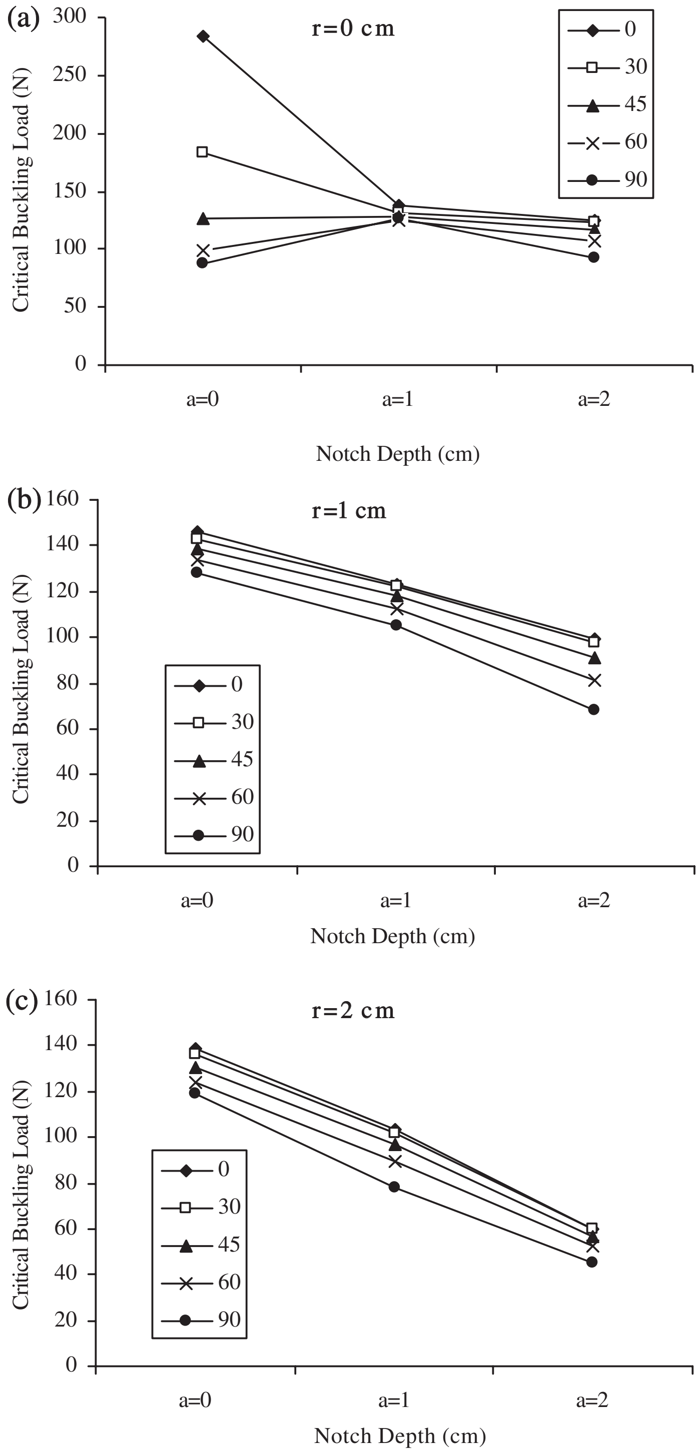
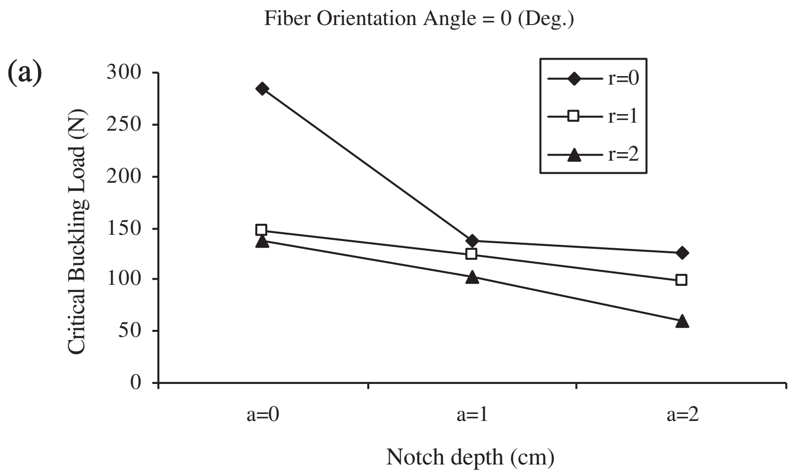


Figure 10. *The critical buckling load variation according to U-shaped cutout depth for (a) r = 0 cm; (b) r = 1 cm; (c) r = 2 cm.*

Table 2. Critical buckling ratios of the E-glass/epoxy composite plates via ply orientation angles.

	P_{cr} (Perforated)/P_{cr} (Unperforated)				
Cutout Size (cm)	**0°**	**30°**	**45°**	**60°**	**90°**
$r=0$, $a=1$	0.48	0.72	1.01	1.28	1.44
$r=0$, $a=2$	0.44	0.67	0.92	1.09	1.06
$r=1$, $a=0$	0.51	0.78	1.09	1.36	1.46
$r=1$, $a=1$	0.43	0.67	0.93	1.14	1.20
$r=1$, $a=2$	0.35	0.53	0.71	0.83	0.78
$r=2$, $a=0$	0.49	0.74	1.03	1.26	1.36
$r=2$, $a=1$	0.36	0.55	0.76	0.91	0.89
$r=2$, $a=2$	0.21	0.33	0.45	0.53	0.52



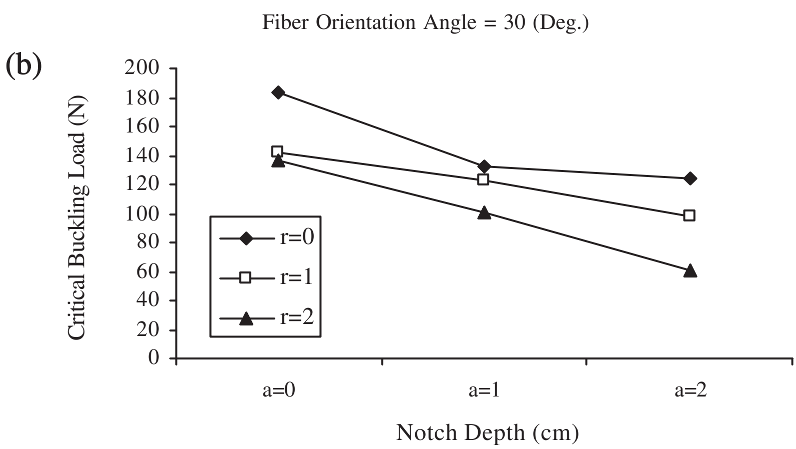
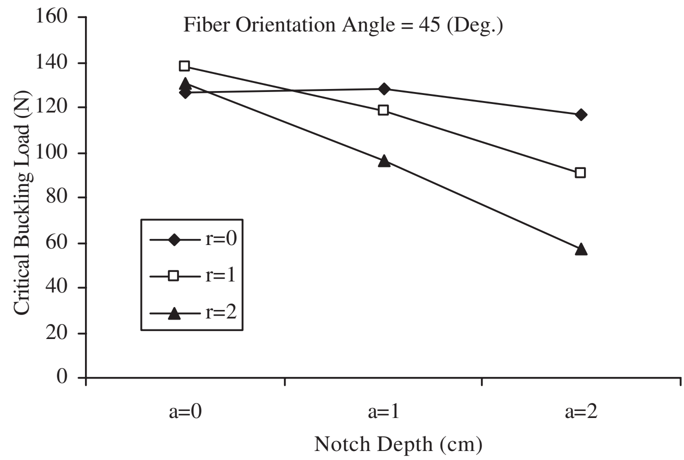


Figure 11. *Variation of critical buckling loads due to U-shaped cutout depth with cutout root radius for fiber orientation angle of (a) $\theta=0$; (b) $\theta=30$; (c) $\theta=45$; (d) $\theta=60$; (e) $\theta=90$.*

(c)



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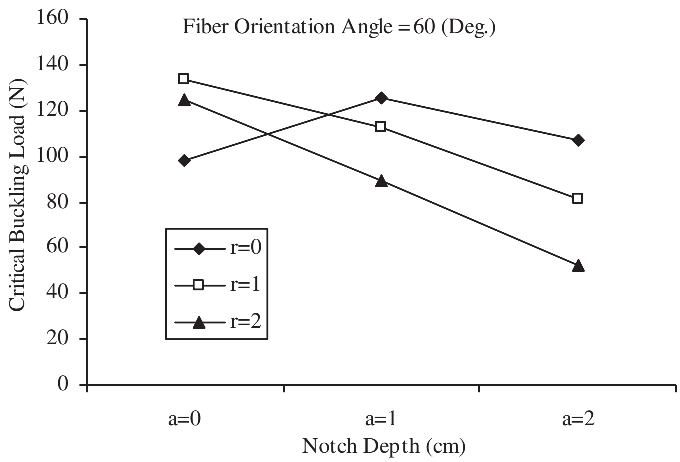

(e)

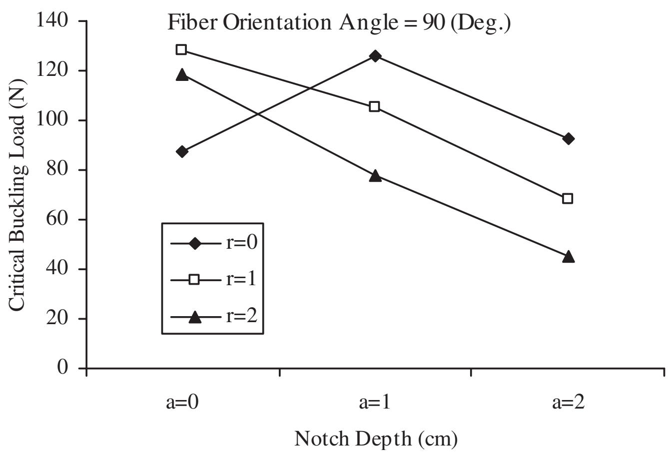

Figure 11. *Continued.*

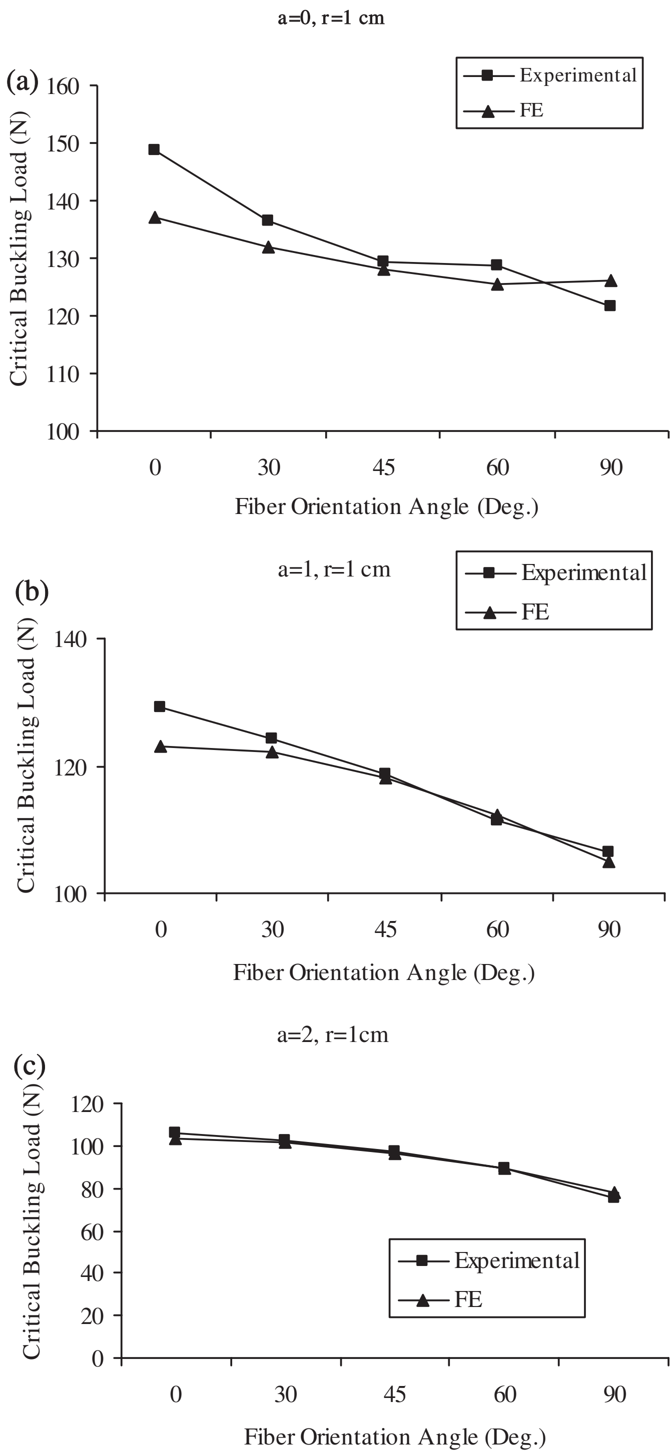

Figure 12. *Comparison of critical buckling response FE vs. experiment for (a) r = 1, a = 0; (b) r = 1, a = 1; (c) r = 1, a = 2.*

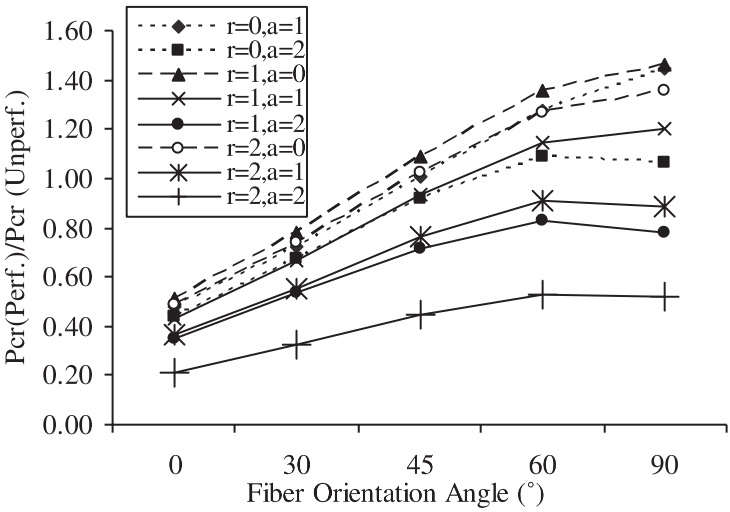

Figure 13. *Comparison of critical buckling ratio for various U-shaped cutouts, based on fiber orientation angle of ply.*

orthotropic plates with mentioned above boundary conditions can not be encountered in any references. The results obtained can be drawn up as follows:

1. The strong dependence of buckling loads on fiber angles has been observed but, the influence of fiber angle on critical buckling loads are decreasing after fiber orientation angle of 45°.
2. The overall effect of cutout shapes is significantly affected on the buckling response of the composite plates.
3. The variation of critical buckling ratios with cutout sizes are put in order from minimum to maximum as $(r=2,\ a=2)<(r=1,\ a=2)<(r=0,\ a=2)<(r=1,\ a=1)<(r=2,\ a=0)<(r=0,\ a=1)<(r=1,\ a=0)$.
4. The effect of notch depth is stronger than that of the notch root radius on buckling loads of plates.
5. Comparison between FE analysis and experimental results are in good agreement with each other and the FE and theoretical buckling responses also show good correlation for all fiber orientation angles.
6. The minimum value of P_{cr}(Perf.)/P_{cr}(Unperf.) ratio (0.21) are obtained for $[0°]_3$ oriented composite plates with maximum cutout size ($a=2$, $r=2$). It means that the buckling capacity of plates is decreased about 79% due to the U-shaped notch.
7. The results indicate that the plates with fiber orientations closest to 0° exhibit the larger reductions in buckling load with increasing cutout sizes. The plates with fiber orientations closest to 90° exhibit very little reduction in buckling loads with increasing cutout sizes and in some cases, no reduction is obtained in buckling loads. The last behavior has been observed in the composite plates having fiber reinforced angles of $\theta \geq 45°$. The maximum critical buckling ratio (1.46) is obtained for $[90°]_3$ fiber reinforced plates perforated with cutout having $r=1$ cm, $a=0$ cm dimensions.
8. In the sharp U-notches ($r=0$) (it can be called crack), the effect of crack depth are decreasing with increasing initial crack length (a). In some cases, the critical buckling

loads exhibit increased values by crack in the composite plates having fiber reinforced angles of $\theta \geq 45^\circ$ with increasing initial crack depth from 1 to 2 mm. This can be important for obtaining critical lateral initial crack size of composite plates under buckling condition.

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