A NUMERICAL ANALYSIS OF A CONVECTIVE STRAIGHT FIN WITH TEMPERATURE-DEPENDENT THERMAL CONDUCTIVITY

by

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In this paper, heat transfer characteristics of a straight fin having temperature--dependent thermal conductivity were computed by using 3-D CFD analysis and MATLAB differential equation solver. The computations were performed with two different cases having both constant and linear function for thermal conductivity property. The CFD and MATLAB results were in good agreement with the data available in the literature. With the help of using these numerical techniques, fin efficiency can be improved and heat transfer rate of fins can be augmented by changing fin materials with variable thermal properties and air-flow conditions. Application of the proposed method can be effectively extended to solve the class of similar non-linear fin problems in engineering and sciences.

Key words: *straight fin, temperature dependent thermal conductivity, fin efficiency*

Introduction

Fins are commonly used in many engineering applications such as air-conditioning systems, chemical processing equipment's, heat exchangers, *etc*. They are generally designed for enhancing heat transfer between base surface and its environment. Convective heat transfer rate can be augmented by different methods such as increasing heat transfer surface area or heat transfer coefficient. Increasing the heat transfer surface area can be achieved by attaching the fins made of highly conductive materials on base surface. Moreover, fin material should have high thermal conductivity to limit the temperature variation from base surface to the tip surfaces of the fin. For convective fins with constant thermal conductivity, temperature distribution of a straight fin requires the solution of linear differential equation [1]. The thermal conductivity depends strongly on the temperature. The temperature-dependent thermal conductivity can be modeled as a linear function of the temperature for many engineering applications [2, 3]. The temperature-dependent thermal conductivity can be significant when the big temperature gradient exists and this causes less/more energy transfer. The governing equation with temperature-dependent thermal conductivity is in the form of non-linear differential equation. The non-linear fin problem has received considerable attention because of its industrial applications such as semi-conductors, heat exchangers, power generators, and electronic components [4]. The non-linear problems with partial differential equations governing the flow are reduced to an ordinary differential equation by similarity transformations. The obtained equations are then solved for the development of series solutions [5]. Some of the

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researches with non-linear differential equations are previously described and the detailed information can be found in available literature. The MHD flow of nanofluid by an exponentially permeable stretching sheet was studied and the effects of different parameters on the velocity and temperature profiles were obtained. The 3-D nanofluid flow and heat transfer in a rotating system in the presence of magnetic field was investigated to determine effects of active parameters on flow, heat and mass transfer [6]. Free convection of ferrofluid in a cavity heated from below in presence of external magnetic field was studied numerically using the Lattice-Boltzmann method [7]. On the other hand, the exact solution can not be obtained for non-linear problem with variable thermal properties in general [8]. So, researchers have focused on using the solution of such non-linear problems by using different semi-analytical or numerical solution methods such as the perturbation method, homotopy perturbation method, variational iteration method, homotopy analysis method, differential transform method Adomian decomposition method, and Akbari-Ganji's method [9-18]. In practical engineering calculations, convective heat transfer coefficient is assumed as constant for all surfaces of the fin but it changes for all surfaces in actual. The CFD tool can be used for calculating the heat transfer coefficient for each surface of a fin. Moreover 3-D CFD simulations can be applied to the fin problem with different flow conditions [19-26]. ANSYS FLUENT software package was used for CFD analysis. ANSYS FLUENT software solves continuum, energy, and transport equations numerically. The detailed information about this software package can be found in [27]. In this study, two different cases (Case 1 and Case 2) were employed for calculating the heat transfer characteristics of a straight fin. For Case 1, the thermal conductivity of fin material was assumed as constant and three different solution methods, (1) Analytical, (2) MATLAB code, and (3) CFD, were applied to the problem. In the solution process, first of all, straight fin problem described in [4] was selected as a test case for the validation of the MATLAB code. Then, the solution of the fin problem was obtained by using different methods abovementioned. For Case 2, the thermal conductivity of a fin material was set as a linear function in numerical calculations and all the other properties were same with Case 1. The numerical results of Case 2 were achieved by using both MATLAB code and CFD method. The solution data were also compared to the each other and non-dimensional temperature function was approximated by a sixth order polynomial series for Case 2. The numerical solutions for the relevant energy balance equation for a prismatic fin in dimensionless variables were achieved by using bvp4c function built in MATLAB software for all cases. The detailed information solving differential equations by using this function can be obtained from [28]. The present study shows that with the help of the using these numerical techniques, fin efficiency and heat transfer rate of fins can be improved by using different fin materials with variable thermal properties and air-flow conditions.

Numerical simulation and method

Modeling geometry and the mathematical definition of the problem

The schematic view of a prismatic fin problem having a constant cross-sectional area, A_c, perimeter, p, and length, b, is shown in fig.1. The base and the ambient temperature are T_b *and* T_a, respectively. The tip surface of the fin is assumed to have an insulated material. Also, the fin is assumed to be thin, which implies that temperature variations in the longitudinal direction are much larger than those in the transverse direction [1]. This assumption can be made when $\mathrm{Bi} \ll 1$ and this means that temperature gradients within the solid are small. The energy equation of a differential element in steady-state conditions is written in eq. (1), Where, h is the convective heat transfer coefficient, and the $k(T)$ is a function of thermal conductivity of fin material. In this equation, the effects of radiation heat transfer and the heat generation are negligible and the h is assumed as uniform for all of the fin surfaces:

$$A_c \frac{\mathrm{d}}{\mathrm{d}x}\left[k(T)\frac{\mathrm{d}T}{\mathrm{d}x}\right] - ph(T - T_\mathrm{a}) = 0 \quad (1)$$

If k is constant and the exact solution of eq. (1) is achieved by the solution of a linear differential equation but if k is approximated by the following relation:

$$k(T) = k_\mathrm{a}[1 + \lambda(T - T_\mathrm{a})] \quad (2)$$

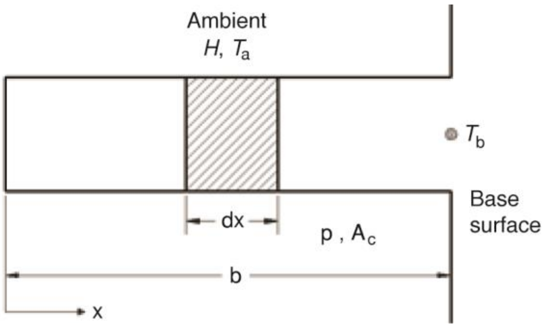


Figure 1. The schematic view of a prismatic fin used in this study

where k_a is the thermal conductivity of the fin at ambient fluid temperature and λ is the parametric definition of the variation of thermal conductivity. The solution of this problem can be obtained from non-linear differential equation solution. Considering temperature-dependent thermal conductivity, non-dimensional quantities can be defined:

$$\theta(x) = \frac{T(x) - T_\mathrm{a}}{T_\mathrm{b} - T_\mathrm{a}}, \quad \xi = \frac{x}{b}, \quad \beta = \lambda(T_\mathrm{b} - T_\mathrm{a}), \quad \psi = \sqrt{\frac{phb^2}{k_\mathrm{a} A_c}} \quad (3)$$

By the using these non-dimensional quantities, the energy equation can be written:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\xi^2} + \beta\theta\frac{\mathrm{d}^2\theta}{\mathrm{d}\xi^2} + \beta\left[\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right]^2 - \psi^2\theta = 0 \quad (4)$$

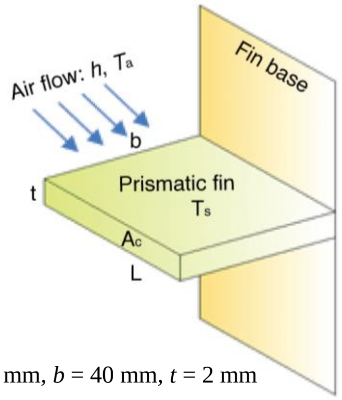
Considering the insulated fin tip problem, for instance, the non-dimensional temperature distribution is assumed to satisfy the following two boundary conditions, eq. (5), and the solution function can be achieved by defining polynomial series for getting the temperature distribution of the fin surfaces:

$$\left.\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right|_{\xi=0} = 0, \quad \theta|_{\xi=1} = 1 \quad (5)$$

Fin efficiency, μ, can be written as stated:

$$\mu = \frac{\int_0^b ph(T - T_\mathrm{a})\mathrm{d}x}{hpb(T_\mathrm{b} - T_\mathrm{a})} = \int_0^1 \theta(\xi)\mathrm{d}\xi \quad (6)$$

The governing equation, eq. (4), with linear temperature-dependent thermal conductivity has non-linear properties. Non-linearity source term is β and when this term is equal to zero, the analytical solution can be found in general form. On the other hand, it is difficult to obtain analytical solutions for all specified functions of thermal conductivity with non-linear terms and to find the temperature distribution of the fin surface is not practical for engineers in general. The dimensions of the prismatic fin used in this study are shown in fig. 2. The thickness of the fin is 2 mm, the length, b, and the width, L, of the fin are 40 mm and 25 mm, respectively.



L = 25 mm, b = 40 mm, t = 2 mm

Figure 2. Dimensions of the prismatic fin used in this study

Mesh structure and boundary conditions for numerical calculations

The computational domain generated for the numerical simulations is shown in fig. 3. The prismatic channel around the fin was created to obtain air-flow characteristics in the computational domain, and the dimension of this channel is 0.25 m (A) x 0.05 m (B) x 0.12 m (C). The effect of the ratio of the L_1 (entrance length) to the L_2 (exit length) on the numerical results were compared and the dimensions were determined ensuring that the air-flow is not affected significantly by the change in the location of the fin in this prismatic channel. The ratio of L_1 to L_2 was determined as 0.25. In this domain hexahedral elements, fig. 4, were used and the number of these elements was about $6 \cdot 10^5$. Mesh should be well designed to resolve important flow features which are dependent upon flow condition parameters. In numerical calculations, mesh generation is very crucial for getting accurate predicted results and reducing computing time. There were a total of six different mesh sizes investigated to determine how fine the mesh should be and to validate the solution independency of the mesh. The predicted averaged fin efficiency for these mesh sizes are listed in tab. 1. The results of fin efficiency was stabilized between $5 \cdot 10^5$ and $6 \cdot 10^5$ cells.

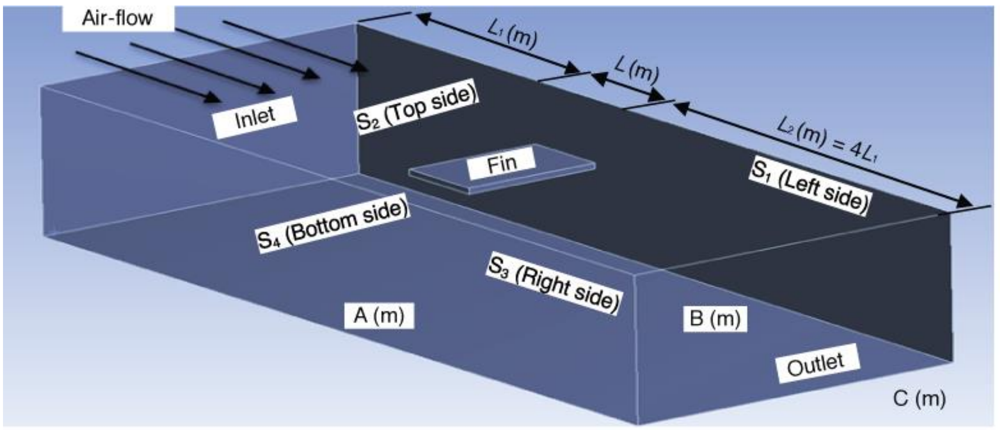


Figure 3. Computational domains for numerical calculations

In numerical calculations, getting precise results are mainly dependent on two factors. These are mesh accuracy and boundary conditions. The air velocity was set to 3 m/s at the inlet surface and air temperature was chosen 30 °C as a constant value at this surface region. At the outlet vent, pressure outlet boundary condition was used. Sides of channel (S_1 to S_4) without inlet and outlet surfaces were considered as adiabatic conditions. The temperature of the base surface of the fin was set to a constant value of 100 °C and tip surface of the fin was set to adiabatic conditions. The material property of fin was selected as steel predefined and referenced in global database built in FLUENT software [22].

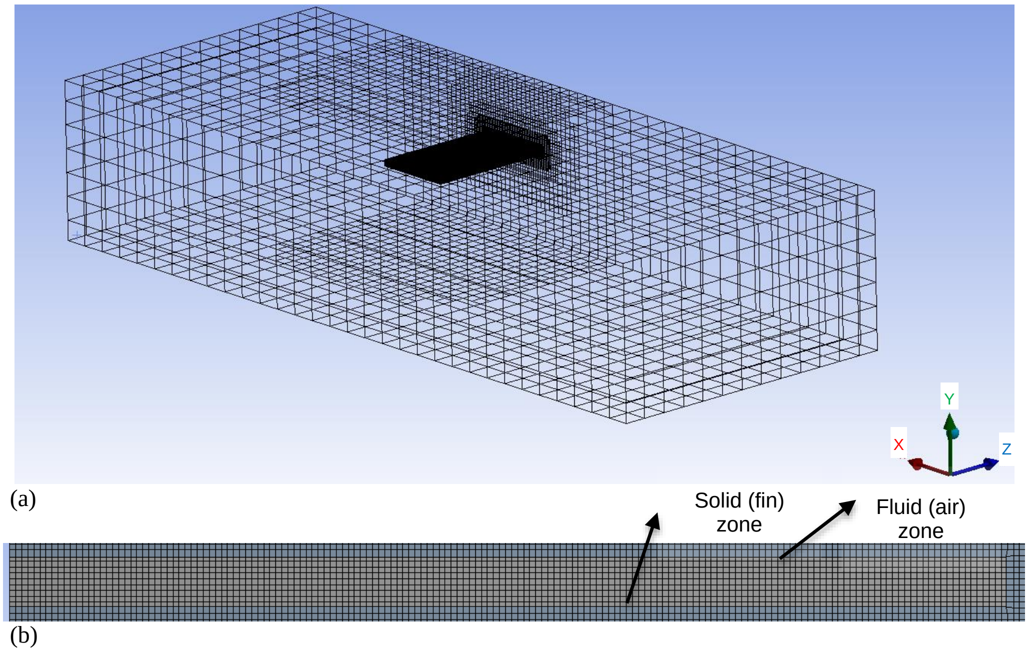


Figure 4. Computational grid (a) 3-D view and (b) the section view of the computational grid in solid region

In the solution process, numerical calculations were performed in steady-state conditions. Air-flow is laminar and SIMPLEC algorithm was chosen for pressure-velocity coupling and cell based option was set for flow field computations. The second order schemes were employed in the discretization of the all governing equations depends on convergence criteria and getting precise results. The convergence criterions were assumed when the normalized residuals of flow equations are less than 10^{-4} and the energy equations are less than 10^{-6}. The governing equations including continuity, momentum, and energy equations for steady-state flow for air region can be written for this problem:

Table 1. Mesh independence test results

Number of cells	Fin efficiency [μ]	
	Case 1	Case 2
50000	38.2	41.3
100000	44.4	48.4
300000	46.8	53.4
500000	47.5	56.4
600000	48.0	57.0
650000	48.2	57.2

– continuity equation

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0 \tag{7}$$

– momentum equations

x-axis direction $$\rho\left(\frac{u\partial u}{\partial x}+\frac{v\partial u}{\partial y}+\frac{w\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\mu_f\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{8}$$

y-axis direction $$\rho\left(\frac{u\partial v}{\partial x}+\frac{v\partial v}{\partial y}+\frac{w\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\mu_f\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{9}$$

z-axis direction $$\rho\left(\frac{u\partial w}{\partial x}+\frac{v\partial w}{\partial y}+\frac{w\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\mu_f\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \tag{10}$$

– energy equation

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=k_\mathrm{f}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{11}$$

where ρ is the density, μ_f – the dynamic viscosity, P – the pressure, k_f – the thermal conductivity of fluid, T – the temperature, c_p – the specific heat, and u, v, and w are velocities of the x-, y- and z-direction, respectively. The energy equation for solid region (fin) can be written:

$$\frac{\partial\left[k(T)\frac{\partial T}{\partial x}\right]}{\partial x}+\frac{\partial\left[k(T)\frac{\partial T}{\partial y}\right]}{\partial y}+\frac{\partial\left[k(T)\frac{\partial T}{\partial z}\right]}{\partial z}=0 \tag{12}$$

Theoretical background for the comparison of numerical results

The analytical solution of the fin problem is described below. In the case of laminar air-flow over the plate, the correlation for Nusselt number can be written as stated in eq. (13), where, Reynolds and Prandtl numbers are dimensionless. h, k, and L are mean heat transfer coefficient at fin surfaces, the thermal conductivity of air at film temperature, and the width of the prismatic fin, respectively. The film temperature is described in eq. (8), where T_s is the mean surface temperature of the fin surfaces along the air-flow and is calculated from CFD results. The air properties for analytical solution were correlated at film temperature, eq. (14), and the schematic view of an air-flow over a plate for the calculation of heat transfer coefficient in this study is shown in fig. 5:

$$\mathrm{Nu}_L=0.664\ \mathrm{Re}_L{}^{0.5}\mathrm{Pr}^{1/3}=\frac{hL}{k_{air}};\qquad \mathrm{Re}_L=\frac{UL}{\nu} \tag{13}$$

$$T_\mathrm{f}=\frac{T_\mathrm{s}+T_\mathrm{a}}{2} \tag{14}$$

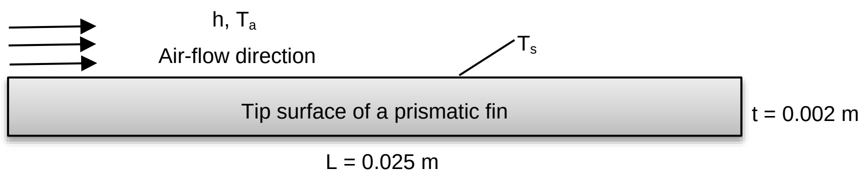


Figure 5. Laminar air-flow over a plate

When the Nusselt number is known and then, theoretical mean heat transfer coefficient can be calculated from eq. (13). After this calculation, theoretical and maximum heat transfer from fin surfaces, $Q_{f,t}$ fin efficiency μ and temperature distribution $T(X)$ can be calculated from eqs. (15)-eq. (18) for insulated fin tip problem, where A_s is the total surface area of a prismatic fin:

$$Q_{f,t} = \left(T_\mathrm{b} - T_\mathrm{a}\right)\sqrt{hpkA_c}\left[\tanh\left(\sqrt{\frac{hp}{kA_c}}L\right)\right] \tag{15}$$

$$Q_{f,max} = hA_s(T_\mathrm{b} - T_\mathrm{a}), \quad A_\mathrm{s}=pb \tag{16}$$

$$\mu = \frac{Q_{f,t}}{Q_{f,max}} \tag{17}$$

$$\frac{T(X) - T_\mathrm{a}}{T_\mathrm{b} - T_\mathrm{a}} = \left\{\frac{\cosh\left[\sqrt{\frac{hp}{kA_c}}(L - X)\right]}{\cosh\left[\sqrt{\frac{hp}{kA_c}}(L)\right]}\right\}, \quad X = L - x \tag{18}$$

Non-dimensional temperature distribution of a straight fin was calculated with three different method for Case 1. The analytical solution was achieved by using correlation for Nusselt number over a plate in laminar flow. The numerical simulation was obtained by using CFD method and MATLAB function defined for solution of boundary value problem for ordinary differential equations. For the numerical solution of eq. (4) subject to eq. (5) is achieved by using function bvp4c in MATLAB. In MATLAB solution, the differential equation described in eq. (4) is solved by defining two new variables as stated in eq. (19). Equation (4) can be rewritten as stated in eq. (21) by using two new variables. After this linearization of the eq. (4), boundary conditions, eq. (5), were used for getting numerical solution of a straight fin problem with temperature dependent thermal conductivity and β was equal to zero for the solution of differential equation for Case 1:

$$\theta = \theta_1 \tag{19}$$

$$\frac{\mathrm{d}\theta_1}{\mathrm{d}\xi} = \theta_2 \tag{20}$$

$$\frac{\mathrm{d}\theta_2}{\mathrm{d}\xi} = \left[\frac{\Psi^2\theta_1 - \beta(\theta_2)^2}{1 + \beta(\theta_1)}\right] \text{for } 0 \le \xi \le 1 \tag{21}$$

The thermo-geometric fin parameter, Ψ, was calculated about 2.12 by using the dimensions of prismatic fin, fig. 2, and the correlation (13) for Nusselt number. In the CFD analysis, the β parameter was selected 0.6 and the λ parameter is calculated about 0.0086 by using eq. (3). From the calculation of these parameters, the thermal conductivity function can be written, eq. (22):

$$k(\mathrm{T}) = 16.27[1 + 0.0086\,(T - 303.15)] \tag{22}$$

The mean heat transfer coefficient was calculated with eq. (23) in FLUENT, where q is heat to the wall from a fluid cell, T_s and T_a are the temperature values of the wall and fluid, respectively:

$$h_f = \frac{q}{T_w - T_\mathrm{f}} \tag{23}$$

Results and discussion

Case 1: Constant thermal conductivity

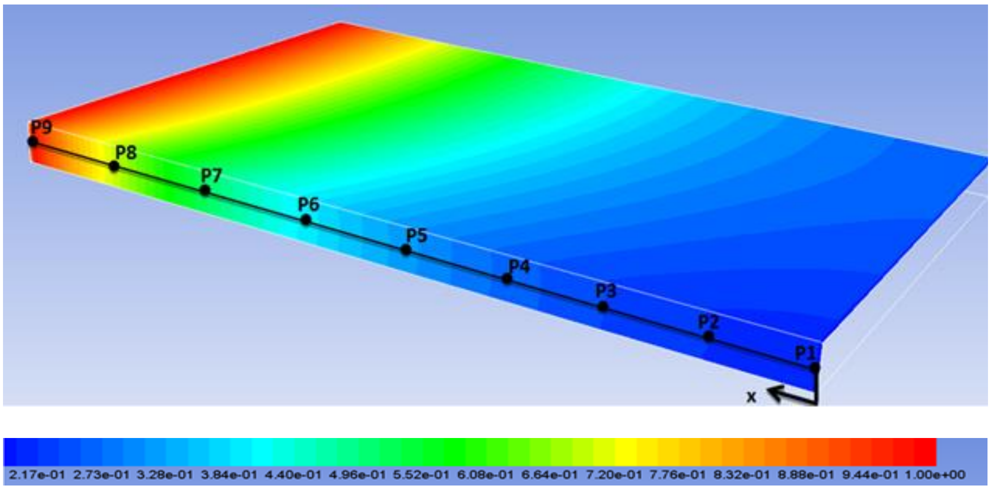
In MATLAB differential equation solver, the thermal conductivity and the thermo-geometric fin parameters were selected 0 and 0.5, respectively. The calculated non-dimensional temperature values are shown in tab. 2. The numerical results are very close to the data obtained from [4]. From the comparison of these results, it can be said that MATLAB code used in this study can be utilized for the solution of the temperature distribution of a straight fin subjected to boundary conditions mentioned in the previous section.

Table 2. Comparison of the data obtained from MATLAB code for Case 1

$\xi\left(\frac{x}{b}\right)$	$\theta(x)=\frac{T(x)-T_a}{T_b-T_a}$ (MATLAB data for $\beta = 0$, $\Psi = 0.5$)	$\theta(x)$ (Data obtained from [2] for $\beta = 0$, $\Psi = 0.5$)
0.0	0.88682	0.88682
0.1	0.88773	0.88793
0.2	0.89090	0.89126
0.3	0.89635	0.89681
0.4	0.90408	0.90461
0.5	0.91411	0.91468
0.6	0.92648	0.92703
0.7	0.94121	0.94169
0.8	0.95835	0.95872
0.9	0.97792	0.97813
1.0	1.00000	1.00000

For the comparison of the CFD results, nine discrete equal spaced points were defined on the symmetric axis of the side surface of the fin. The non-dimensional temperature values obtained from three different methods for Case 1 was shown in fig. 6. The numerical results obtained in Case 1 have same trend with exact solution.

Computed heat transfer characteristics of a straight fin by using dimensions and related equations were shown in tab. 3 for Case 1. In analytical calculations, the maximum heat transfer were computed as a value of 5.95 W and the theoretical heat transfer from the fin surfaces was computed about 2.94 W by using eqs. (15)-(18). In the analytical solution, mean heat transfer coefficient was calculated by using correlation over a flat plate with laminar air-flow. But for this calculation, the surface temperature of a prismatic fin was required. This value was taken from the results of CFD analysis for the computation of film temperature and the other necessary parameters. The mean surface temperature of a fin was calculated about 63.02 °C and the film temperature was calculated about 46.51 °C by using eq. (14). The mean heat transfer coefficient was calculated about 42.49 W/m^{2}K by using analytical method. Fin efficiency of a straight fin was calculated about 49.4 % in analytical solution. In CFD analysis, this value was calculated about 48% and the mean heat transfer coefficient was computed as a value of 41.90 W/m^{2}K by using CFD results and eq. (23). The temperature of the tip surface of the fin was computed about 47 °C in both analytical and CFD methods for Case 1.



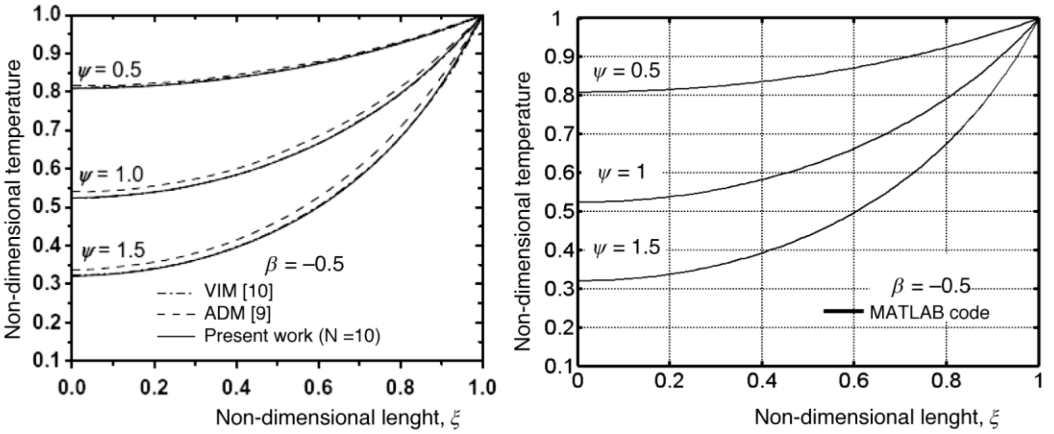
Points	x(m)	$\xi\left(\frac{x}{b}\right)$	$\theta(x)$(CFD)	$\theta(x)$ MATLAB $\beta = 0$, $\Psi = 2.124$	$\theta(x)$ Exact solution
P1	0.000	0.000	0.217	0.236	0.236
P2	0.005	0.125	0.235	0.244	0.244
P3	0.010	0.250	0.254	0.270	0.269
P4	0.015	0.375	0.273	0.315	0.314
P5	0.020	0.500	0.328	0.382	0.382
P6	0.025	0.625	0.403	0.476	0.476
P7	0.030	0.750	0.589	0.604	0.604
P8	0.035	0.875	0.720	0.774	0.774
P9	0.040	1.000	1.000	1.000	1.000

Figure 6. Comparison of computed non-dimensional temperature values at discrete nine points with different methods for Case 1

Table 3. Computed heat transfer characteristics of a straight fin for Case 1

Fin properties for Case 1				Heat transfer characteristics of fin		
					Calculated (analytical)	Calculated (CFD)
Parameter	Value	Parameter	Value	$Q_{f,max}$ [W]	5.95	5.87
A_c [m^2]	0.0005	p [m]	0.054	$Q_{f,t}$ [W]	2.94	2.81
A_s [m^2]	0.0020	L [m]	0.040	h [Wm^{-2}K^{-1}]	42.49	41.90
k_a [Wm^{-1}K^{-1}]	16.27	T_a [°C]	30	μ [%]	49.41	48.0
T_f [°C]	46.51	T_b [°C]	100	T_{tip} [°C]; ($x=0$)	46.5 °C	47.1 °C

For the comparison of the numerical results of temperature-dependent thermal conductivity fin problem by using MATLAB function for Case 2, non-dimensional temperature data obtained from [4] was used. From the results of numerical data shown in fig. 7, the numerical solution data, fig. 7(b), obtained from MATLAB differential equation solver have same trend with the values of non-dimensional temperature parameter data, fig. 7(a), shown in [4]. The computed values were good agreement with the data for different thermo-geometric fin parameters.



(a) (b)

Figure 7. (a) Non-dimensional temperature distribution obtained by using different numerical methods shown in [4] and (b) non-dimensional temperature distribution obtained in this study by using MATLAB code

Table 4. Computed heat transfer characteristics of a straight fin for Case 2

Characteristics	Calculated values MATLAB β = 0.6, Ψ = 2.124	Calculated values (CFD)
Q_t [W]	3.15	3.26
$Q_{f,max}$ [W]	5.95	5.70
h [Wm^{-2}K^{-1}]	42.49	40.74
μ [%]	53	57

The fin efficiency was calculated about 53% for Case 2. This value was calculated about 57% obtained from the numerical results by using CFD method. Computed heat transfer characteristics of a straight fin for Case 2 was shown in tab. 4. The total heat transfer from the all fin surfaces was calculated about 3.2 W by using MATLAB solver and this value was calculated about 3.3 W in the CFD analysis for Case 2. In Case 2, the thermal conductivity parameter was selected as a positive value of 0.6 and as expected, the fin efficiency was calculated higher than the value obtained from Case 1. The fin efficiency was found about 53% from the MATLAB solver and 57% from the CFD results. The mean heat transfer coefficient was computed about 41 W/m^{2}K by using CFD method and this value was calculated about 42 W/m^{2}K for Case 1. From these calculations, mean heat transfer coefficients were computed approximately close together for two cases in CFD method and this fulfill the assumption as the constant mean heat transfer coefficient between Case 1 and Case 2. Fin efficiency was computed with MATLAB solution data for different thermo-geometric fin and thermal conductivity parameters by using eq. (6). The computed value of fin efficiency with MATLAB was shown in fig. 8(a). The fin efficiency is increased with the raising of thermal conductivity parameter for a constant value of thermo-geometric fin parameter. But the fin efficiency was decreased with the rising of thermo-geometric fin parameter for a constant value of thermal conductivity parameter. So, it can be easily said that for the designing of more efficient straight fins, if possible we have to select the lower thermo-geometric fin and the higher thermal conductivity parameter. Another important result is that the cases with variable thermal conductivity have asymmetric solution data $\beta = -0.8, -0.6, 0, 0.6, 0.8)$ according to curve obtained from exact $\beta = 0$ solution.

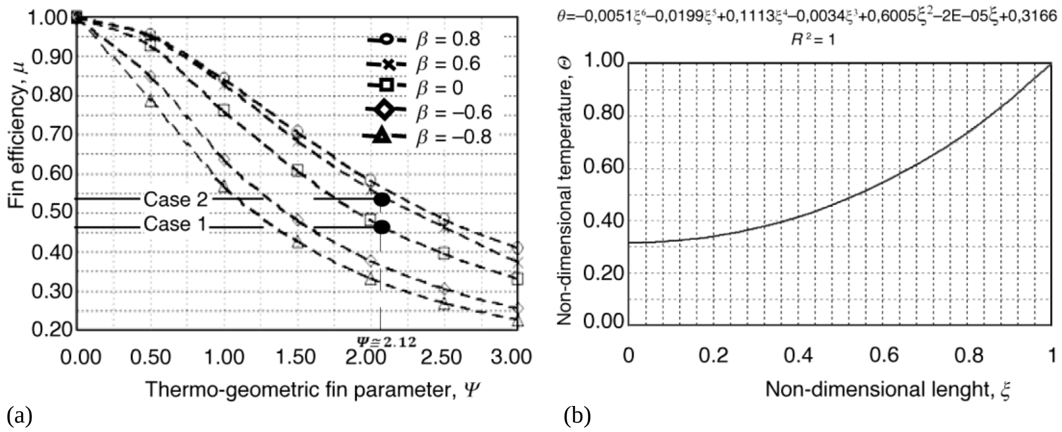
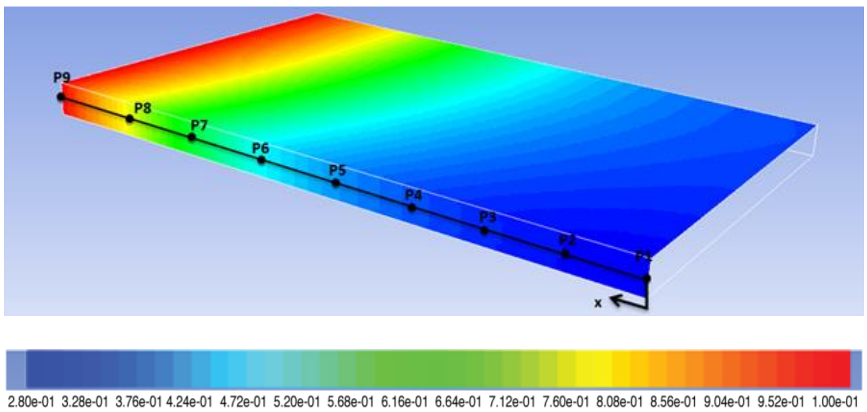


Figure 8. (a) Calculated fin efficiency with thermo-geometric fin parameter and (b) non-dimensional temperature depends on non-dimensional length (Case 2: β = 0.6, Ψ 2.124)



Points	x(m)	$\xi\left(\frac{x}{b}\right)$	$\theta(x)$ CFD	$\theta(x)$ MATLAB β = 0.6, Ψ = 2.124
P1	0.000	0.000	0.280	0.317
P2	0.005	0.125	0.324	0.326
P3	0.010	0.250	0.345	0.355
P4	0.015	0.375	0.376	0.403
P5	0.020	0.500	0.450	0.473
P6	0.025	0.625	0.545	0.565
P7	0.030	0.750	0.664	0.683
P8	0.035	0.875	0.776	0.827
P9	0.040	1.000	1.000	1.000

Figure 9. Comparison of computed non-dimensional temperature values at discrete 9 points with different methods for Case 2

The MATLAB solution data shown in fig. 8(b) and to calculate the fin efficiency eq. (6) were used. The sixth order polynomial function was used to represent the non-dimensional temperature depends on non-dimensional length. The R^2 value was equal to one. The computed non--dimensional temperature values for Case 2 were shown in fig. 9 and the CFD results were approximately close to the MATLAB solution data for Case 2.

Conclusions

In this paper, heat transfer characteristics of a straight fin having temperature-dependent thermal conductivity were computed. In contrast with the conventional numerical techniques used to solve the non-linear differential equations, the proposed solution method contains both CFD and MATLAB solver. The computations were performed with two different cases having both constant and linear function for thermal conductivity property. The temperature distribution parameterized by two fin parameters (β, Ψ) was obtained for different conditions. One of the important results is that the cases with temperature dependent thermal conductivity have asymmetric solution curves data according to exact solution.

Another result obtained from this study is that the 1-D and 3-D numerical solution methods can be easily applied together to utilize unique advantages of these methods. For instance, in 3-D CFD solutions, the local and mean heat transfer coefficient can be computed for a specified fin-geometry for different conditions. On the other hand, we can easily determine the effects of different thermal conductivity parameter on the fin efficiency in 1-D solution approach. By the using of these numerical techniques together, the fin efficiency can be evaluated and augmented by selecting different fin materials with variable thermal properties. Application of these numerical techniques can be extended to solve the fin problems with different geometrical fin shape such as parabolic, triangular, cylindrical, and *etc*. The CFD results were in good agreement with the data obtained from the MATLAB differential equation solver and the data available in the literature.

Nomenclature

A_c – cross-section area of a prismatic fin, [m^2]
A_s – total surface area of a prismatic fin, [m^2]
b – length of a fin, [m]
c_p – specific heat capacity, [Jkg^{-1}K^{-1}]
h – convective heat transfer coefficient, [Wm^{-2} K^{-1}]
h_f – computed convective heat transfer coefficient by using CFD, [Wm^{-2} K^{-1}]
k – thermal conductivity of fin material, [Wm^{-1}K^{-1}]
k_a – thermal conductivity of fin material at ambient fluid temperature, [Wm^{-1}K^{-1}]
k_f – thermal conductivity of fluid, [Wm^{-1}K^{-}]
L – width of a prismatic fin, [m]
Nu_L – Nusselt number, (=hL/k), [–]
P – pressure, [Pa]
p – perimeter of cross-section area of a prismatic fin, [m]
q – computed heat flux value for fin surfaces by using CFD, [Wm^{-2}]
$Q_{f,max}$ – maximum heat transfer from fin surfaces, [W]
$Q_{f,t}$ – theoretical heat transfer from fin surfaces, [W]
Q_t – total heat transfer from fin surfaces, [W]
P_R – Prandtl number, (= ν/α), [–]
Re_L – Reynolds number, (= UL/ν), [–]
T_a – air temperature, [K, °C]
T_b – temperature of a base surface of a prismatic fin, [K, °C]
T_{tip} – temperature of a tip surface of a prismatic fin, [K, °C]
T_f – film temperature, [K, °C]
T_s – surface temperature of a prismatic fin, [K, °C]
t – thickness of a prismatic fin, [m]
U – air velocity, [ms^{-1}]

Greek symbols

β – thermal conductivity parameter
$\theta(x)$ – non-dimensional quantity for temperature distribution
λ – parametric definition of the variation of thermal conductivity
μ – fin efficiency
μ_f – dynamic viscosity of air, [kgm^{-1}s^{-1}]
ν – kinematic viscosity of air, [m^{2}s^{-1}]
ξ – non-dimensional quantity for length (b) of a prismatic fin
ρ – density, [kgm^{-3}]
ψ – thermo-geometric fin parameter

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