Volume 3, No. 1, 1989

Journal of the Faculties of
Engineering of Uludağ University

A STUDY ON RADIAL DEFORMATION IN JOURNAL BEARING

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ABSTRACT

In this study, aiming to achieve a more accurate analyse of performance radial journal bearings, the effects of shaft and liner deformations have been considered together. If length/diameter ratio is quite small, the results given confirm well with those obtained in practice. When this ratio is high, however, deteriotaritons in the shaft deflection or linersgeometry will be more effective than expected radial deformations.

ÖZET

Radyal Kaymalı Yataklarda Deformasyonların Hesabı

Bu çalışma, radyal kaymalı yatakların performans karakteristiklerini, elastik etkileri de dikkate alarak hesaplamayı hedef almaktadır.

Rijid kabul ve sadece burcun elastik kabulüyle bulunan performansların yanında, bu çalışma milin de elastik olduğu gerçek durumdaki performansları en iyi şekilde verebilmektedir.

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1. INTRODUCTION

When hydrodynamic conditions have been achieved in radial journal bearings an oil film capable of carrying bearing load forms itself. Hydrodynamic conditions are determined by the relative velocity between sliding surfaces and a cross section with an area restricted in the direction of motion.

Carl[1], in his experiments in 1964, proved that pressure of the oil film, p (θ, z), causes deformation in bearing and it is understood that for metals at about 2000 1 b/in^2 (~ 140 at) this deformation is seriously important. Carl also showed how bearing performance characteristics are influenced by these deformations.

Common feature of oil studies in this field is that they all consider liner deformations while shaft deformations are ignored. In this study, the effect of shaft deformation in bearing performance characteristics, besides that of liner deformation, is investigated. It is expected and found out that an account of the effect of shaft deformation results with a considerable in these characteristics.

2. ASSUMPTIONS

1. Reynolds equation is solved with Reynolds boundary conditions. These are:

$$\left[\begin{array}{ll} \theta = \theta_1 = 0, & p = 0 \\ \theta = \theta_2, & p = 0 \quad \frac{\partial p}{\partial \theta}\Big|_{\theta_2} = 0 \end{array}\right] \text{ in } \theta\text{-direction}$$

$$\left[\begin{array}{ll} z = 0 & \frac{\partial p}{\partial z} = 0 \\ z = \mp 1/2 & p = 0 \end{array}\right] \text{ in z-direction}$$

2. Liner is fitted to the body tightly so that no deformation is observed along the outer periphery of the liner, i.e. u = v = w = 0. In addition to this liner is not allowed for axial motion (w = 0).

3. Elastic deformation of bearing is assumed to be caused by oil pressure without any deflection. Therefore l/d ratios of 0.5 and 1.0 are used.

4. Temperature is taken to be constant. This assumption results with $\eta = \eta(p)$.

5. During the runs shaft and liner axis are parallel and deformations along the shaft axis are determined so as to become finite. Also the relative po-

sition of the shaft and bearing at the end surfaces assumed to survive during the experiment (i.e. the relative position of the shaft doesn't change at any end during the experiments).

6. Elastisity theory is utilized for calculation of deformations and solution is based on strain equation and equations of motion.

7. The effects of liner thickness, liner materials and variable viscosity in deformation are also considered in the calculation.

3. NONDIMENSIONALIZATION[2]

It is quite useful to use nondimensional parameters in order to avoid the calculational difficulties due to abundance of variables. Some nondimensional parameters used in this study are as follows:

$$p = \frac{n_{C} \, . \, u \, . \, R}{c^2} \, . \, P$$

$$h = c \; H$$

$$G = \frac{\eta}{\eta_o} = e^{\alpha P}$$

$$z = z.R$$

Using these nondimensional parameters the Reynolds equation:

$$\frac{1}{R^2} \frac{\partial}{\partial \theta} (h^3 \frac{\partial p}{\partial \theta}) + \frac{\partial}{\partial z} (h^3 \frac{\partial p}{\partial z}) = 6\eta \frac{1}{R} u \frac{\partial h}{\partial \theta} \tag{3.1}$$

takes the following nondimensional form:

$$\frac{\partial}{\partial \theta} (\frac{H^3}{G} \frac{\partial p}{\partial \theta}) + \frac{\partial}{\partial z} (\frac{H^3}{G} \frac{\partial P}{\partial z}) = 6 \frac{\partial H}{\partial \theta} \tag{3.2}$$

Equation can be reduced to a finite-defference or a finite-element form so that it can be solved easily by a numerical iterative technique. In each iteration step the equations are solved as many times as the number of grids and pressure distribution is determined.

4. BEARING DEFORMATION ANALYSIS[3]

For deformation analysis, we will start with component of the strain tensor which is well-known for compresible fluid flow computations and analysis of linear elastic materials. This is given by:

$$t_{k1} = \lambda e_{ii} \delta_{k1} + 2\mu e_{k1} \tag{4.1}$$

where λ and μ are tame constants which are defined in terms of young modulus and poisson's ratio as.

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

$$\mu = \frac{E}{2(1+\nu)}$$

δ_{k1} = Kronecker's delta

e_{k1} = Linearized strain tensor

$\overline{u}$ (r, θ, z); $\overline{v}$ (r, θ, z) and $\overline{w}$ (r, θ, z) being the radial, tangential and axial deformation vectors respectively, components of deformation tensor can be expressed by:

$$e_{k1} = \frac{1}{2}(\overline{u}_{k;1} + \overline{u}_{1;k})$$

In cylindirical coordinates, the components of infinitesinual strain tensor are defined by.

$$e_{rr} = \frac{\partial \overline{u}}{\partial r}$$

$$e_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial \overline{u}}{\partial \theta} + \frac{\partial \overline{v}}{\partial r} - \frac{\overline{v}}{r}\right)$$

$$e_{rz} = \frac{1}{2}\frac{\partial \overline{u}}{\partial z}$$

$$e_{\theta\theta} = \frac{1}{r}\frac{\partial \overline{v}}{\partial \theta} + \frac{\overline{u}}{r}$$

$$e_{\theta z} = \frac{1}{2}\frac{\partial \overline{v}}{\partial z}$$

$$e_{zz} = \frac{\partial \overline{w}}{\partial z}$$

By using these, components of stress tensor are obtained as follows:

$$t_{rr} = (\lambda + 2\mu)e_{rr} + \lambda(e_{\theta\theta} + e_{zz})$$

$$t_{r\theta} = 2\mu e_{r\theta}$$

$$t_{rz} = 2\mu e_{rz}$$

$$t_{\theta\theta} = (\lambda + 2\mu)e_{\theta\theta} + \lambda(e_{rr} + e_{zz})$$

$$t_{\theta z} = 2\mu e_{\theta z}$$

$$t_{zz} = (\lambda + 2\mu)e_{zz} + \lambda(e_{rr} + e_{\theta\theta})$$

Second step in the analysis is the introduction of equaiton of motion. In it's general form, the equation can be written as:

$$t_{k1;k} + \rho f_1 = \rho \ddot{u}_1 \tag{4.2}$$

If the body forces and the deformations are static, this equation takes its simple form.

$$t_{k1;k} = 0 \tag{4.3}$$

Introducing the proper derivatives from stress components in the equation above, deformation equations can be obtained.

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial r^2} + \frac{\mu}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \mu\frac{\partial^2 u}{\partial z^2} + (\lambda + 2\mu)\frac{1}{r}\frac{\partial u}{\partial r} - (\lambda + 2\mu)\frac{u}{r^2}$$

$$-\frac{1}{r^2}\frac{\partial v}{\partial \theta}(\lambda + 3\mu) + \frac{1}{r}\frac{\partial^2 v}{\partial r \partial \theta}(\lambda + 2\mu) + \lambda\frac{\partial^2 w}{\partial r \partial z} = 0$$

$$\mu\frac{\partial^2 v}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}(\lambda+2\mu) + \mu\frac{\partial^2 v}{\partial z^2} + \mu\frac{1}{r}\frac{\partial v}{\partial r} - \mu\frac{v}{r^2} + (\lambda+3\mu)\frac{1}{r^2}\frac{\partial u}{\partial \theta} \tag{4.4}$$

$$+ (\lambda+\mu)\frac{1}{r}\frac{\partial^2 u}{\partial r \partial \theta} + \frac{1}{r}\lambda\frac{\partial^2 w}{\partial \theta \partial z} = 0$$

$$(\lambda+2\mu)\frac{\partial^2 w}{\partial z^2} + \frac{1}{r}\frac{\partial u}{\partial z}(\lambda+\mu) + (\lambda+\mu)\frac{\partial^2 u}{\partial r \partial z} + (\lambda+\mu)\frac{1}{r}\frac{\partial^2 v}{\partial z \partial \theta} = 0$$

Series solutions of these equations are as follows:[4]

$$\overline{u} = a\, u(r) \operatorname{Cos}\frac{kz}{a} \operatorname{Cos} n\theta$$

$$\overline{v} = a\, n\, v(r) \operatorname{Cos}\frac{kz}{a} \operatorname{Sin} n\theta \tag{4.5}$$

$$\overline{w} = a\, k\, w(r) \operatorname{Sin}\frac{kz}{a} \operatorname{Cos} n\theta$$

Here u(r), v(r) and w(r) are the functions to be determined. Let us make change of variables as defined by $c = 2 + \lambda/\mu$ and $y = r/a$. Inmserting the general solutions (4.5) in equation (4.4) with the new variables, a simpler form of equations of motion can be obtained:

$$c\frac{d^2u}{dy^2}+\frac{c}{y}\frac{du}{dy}-(n^2+k^2y^2+c)\frac{u}{y^2}+\frac{n^2}{y}(c-1)\frac{dv}{dy}$$

$$-(c-1)\frac{n^2}{y^2}v+k^2(c-1)\frac{dw}{dy}=0$$

$$\frac{d^2v}{dy^2}+\frac{1}{y}\frac{dv}{dy}-\frac{v}{y^2}(n+n^2c+k^2y^2)-\frac{1}{y}(c-1)\frac{du}{dy}$$

$$-\frac{u}{y^2}(c+1)-\frac{k^2}{y}(c-1)w=0 \qquad (4.6)$$

$$\frac{dw}{dy^2}+\frac{1}{y}\frac{dw}{dy}-(\frac{n}{y^2}+ck^2)w-(c-1)\frac{du}{dy}-\frac{u}{y}(c-1)-\frac{v}{y}n^2(c-1)=0$$

It is obvious that deformation components which are functions of r only by this last set of equation, can be calculated easily.

5. BOUNDARY CONDITIONS

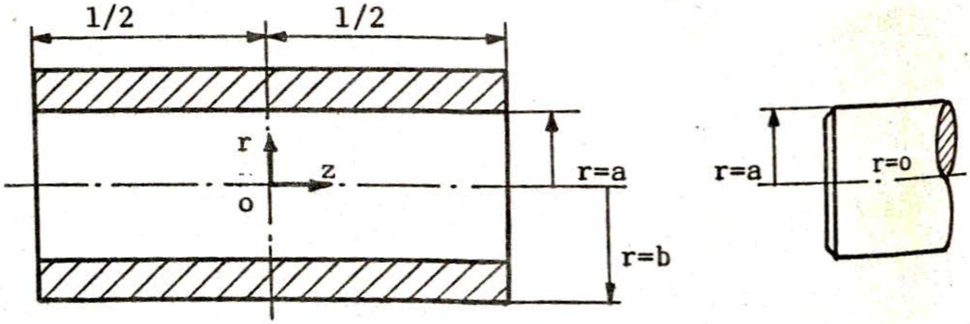

Fig. 1 - Dimensions of the journal bearing

– For the inner surface of the liner (r = a), Fig. 1.

$$c\frac{du}{dy}=-\frac{1}{\mu}-(c-2)\frac{n^2v}{y}+\frac{u}{y}+k^2w \qquad \text{for } t_{rr}=-p$$

$$\frac{dv}{dy}=\frac{u}{y}-\frac{v}{y} \qquad \text{for } t_{r\theta}=0$$

$$\frac{dw}{dy}=0 \qquad \text{for } t_{rz}=0$$

– For the outer surface of the liner ($r = b$)

$$u = v = w = 0$$

Also both ends of the bearing are free of motion:

For $r = b$ at $z = 0$ and $z = \pm 1/2$

$$w = 0$$

$$t_{r\theta} = 0$$

$$t_{rz} = 0$$

For the shaft at $r = a$ the above conditions are valid while at $r = 0$ the equality

$$u = v = w = \text{finite}^5$$

must hold.

With these boundary conditions, the set of differential equations can be solved by using either Bessel functions or applying the step-by-step integration of Runge-Kutta approximation as it is done here.

6. METHOD OF SOLUTION

1. Pressure distribution in bearings is found by using the finite-difference method applied to Reynolds equation of differential form.

2. The equation for pressure distribution is expressed in the form of double Fourier series, since this form allows an easy calculation of deformation.

$$p = \sum_m \sum_n p_{m,n} \operatorname{Cos} \frac{m\P z}{1} \operatorname{Cos} (n\theta + \beta_{m.n})$$

3. In the case of unit dimensionless pressure amplitudes ($p_{m,n} = 1$), pressure distrubution belonging to any point P(m,n) is in the form

$$p = \operatorname{Cos} \frac{kz}{a} \operatorname{Cos} n\theta \qquad (k = \frac{2m\pi a}{1})$$

since pressure is not under the effect of phase difference angle $\beta_{m,n}$. In that case, total deformation value at any point of the bearing can be found by using the calculated deformation coefficients u(r), v(r), w(r);

$$u = a\, u(r) \operatorname{Cos} \frac{kz}{a} \operatorname{Cos} n\theta$$

$$v = a\, n\, v(r) \operatorname{Cos} \frac{kz}{a} \operatorname{Sin} n\theta$$

$$w = a\, k\, w(r) \operatorname{Sin} \frac{kz}{a} \operatorname{Cos} n\theta$$

When these deformations multiplied by the pressure amplitudes $P_{m,n}$, corresponding to the points P(m,n), deformation value corresponding to this term can be found. Summation over m and n gives the overall deformation of any point.

$$u = \sum_m \sum_n p_{m,n}\, u(r)_{m,n} \,\mathrm{Cos}\,\frac{m\pi z}{1}\,\mathrm{Cos}\,(n\theta + \beta_{m,n})$$

$$v = \sum_m \sum_n p_{m,n}\, v(r)_{m,n} \,\mathrm{Cos}\,\frac{k\pi z}{a}\,\mathrm{Sin}\,(n\theta + \beta_{m,n}) \qquad (6.1)$$

$$w = \sum_m \sum_n p_{m,n}\, w(r)_{m,n} \,\mathrm{Sin}\,\frac{k\pi z}{a}\,\mathrm{Cos}\,(n\theta + \beta_{m,n})$$

If the total deformations obtained in this manner are added to the initial oil film thickness, actual film thicknesses for every point can easily be found.

Performance characteristics of the journal bearings with rigid lineir and two elastic liners of steel and brass are compared for constant load of w = 5.6 kN, including shaft deformation, in Table 1. In Table 2 variation of performance characteristics with viscosity is given.

Table 1. Performance Characteristics at Constant Loading Case (W = 5.6 Kn)

	ψ	ε	H_{min}	ϕ	N	f^*	
1.	0.00	0.63	0.37	47°	1.15	2.72	bearing with rigid liner
2.	0.04	0.66	0.35	43°	1.07	2.90	bearing with steel liner
3.	0.08	0.69	0.33	41°	1.05	3.09	bearing with brass liner

Table 2. Performance Characteristics (ε = 0.5, l/d = 1.0, material steel)

c	P_{max}	w	ϕ	H_{min}	ε	α'
0.018	1.879	3.22	33.18	0.512	0.487	0.00
0.018	1.976	3.32	33.48	0.512	0.487	0.05
0.018	2.084	3.43	33.74	0.512	0.487	0.1

7. CONCLUSIONS

1. Unlike the results obtained with rigid bearings, modulus of elasticity and liner thickness are found to be effecting the bearing performance.

2. Eccentricity ratio, which was constant in rigid bearings, increased with deformation.

3. With increased deformation, pressure distribution showed improved uniformity, maximum pressure area increased and maximum pressure decreased.

4. Load angle decreased due to deformation. This results with a better stability in journal bearing systems.

5. Based on the result that minimum film thickness increases with deformation, it can be concluded that it is possible to use eccentricity ratios greater than unity in elastic journal bearings without any metal-to-metal contact between liner and the shaft (i.e liquid friction still survives).

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NOMENCLATURE

c	radial clearance
C	a costant
d	bearings diameter
E	Young's modulus of bearing lineer and shaft
f^*	friction coefficient parameter
h	fluid film thickness
H	nondimensional fluid film thickness
H_{min}	minimum film thickness
l	bearing length in z-direction
N	power loss
P	pressure
P	nondimensional pressure

R	journal radius
t	bearing lineer thickness
t_0	ratio of lineer thickness to inner radius of the bearing lineer
u	journal velocity
u,v,w	components of bearing deformation
$\bar{u},\bar{v},\bar{w}$	nondimensional of bearing deformation
w	load capacity
α	piezoviscous coefficient
u	poisson's ratio
ε	eccentricity ratio
ε^*	new eccentricity ratio
η	viscosity of the lubricant
η_0	viscosity at atmosferic pressure
ϕ	attitude angle
ψ	nondimensional deformation coefficient
r,θ,z	cylindrical coordinates