DE GRUYTER
OPEN

*Journal of* ELECTRICAL ENGINEERING, VOL 67 (2016), NO6, 444–448

# THE NEW FIELD QUANTITIES AND THE POYNTING THEOREM IN MATERIAL MEDIUM WITH MAGNETIC MONOPOLES

**Ömer Zor** *

The duality transformation was used to define the polarization mechanisms that arise from magnetic monopoles. Then, a dimensional analysis was conducted to describe the displacement and magnetic intensity vectors (constitutive equations) in SI units. Finally, symmetric Maxwell equations in a material medium with new field quantities were introduced. Hence, the Lorentz force and the Poynting theorem were defined with these new field quantities, and many possible definitions of them were constructed.

K e y w o r d s: magnetic monopoles, Maxwell equations, Lorentz force, Poynting theorem

## 1 INTRODUCTION

Maxwell [1] developed his equations based on the assumption of no magnetic charges. The equations, however, suggested the concept of magnetic charges. Heaviside [2] first observed that the Maxwell equations are invariant in vacuum under the electromagnetic duality transformations. He thought that there should be magnetic charges and currents in Maxwell equations, although there are no experimental results that show magnetic monopoles. Larmor [3] considered and generalized Heaviside's discrete transformations as continuous rotations of a complex field. Larmor's formulation served continuous freedom in the choice of electric and magnetic fields as the radiation fields. The magnetic monopole has received increasing interest, as its existence is not forbidden by any known principles of physics.

Electromagnetic duality exists only for 3 spatial dimensions. This does not mean it can't be by physically correct for 3 spatial dimensions, but it remains that electromagnetic duality is something of a theoretical oddity [4]. It should be stated that, the symmetric form of the Maxwell equations facilitates the solutions of many problems in radiation and scattering.

Poincaré [5] studied the classical dynamics of a moving electron interacting with the field of a fixed magnetic monopole and gave angular momentum equation. Poincaré's works can be concluded as involving intrinsic angular momentum, nevertheless Poincaré was not aware of this. Thomson [6] later reported the same results and defined the intrinsic angular momentum in resulting equation. Multiplication of the radial component of the conserved orbital angular momentum by half of the modified Planck constant results in a quantization condition. However, Dirac [7,8] first developed a quantization condition such that the unobservability of phase in quantum mechanics allows singularities as sources of magnetic fields, which is similar to point-like electric charges as sources of electric fields. Thus, Dirac showed that the electric vector potential must be singular in the presence of a magnetic monopole. The singularity occurs on a line instantaneously extending outward from the monopole to spatial infinity. Schwinger [9–12] generalized this quantization condition to dyons (particles with an electric and magnetic charge). Later, Schwinger attempted to construct a manifestly consistent field theory of dyons but was unsuccessful.

In the works of Dirac, Schwinger and Zwanziger [13], it was shown that it is not possible to develop an electromagnetic theory of point-like electric and magnetic sources without introducing the Dirac string or multivalued potential [14].

In addition to these works, Jackson [15] analyzed the dynamics of subatomic particles (electron, muon, proton, neutron, nuclei) and showed the intrinsic magnetic moments of particles to be caused by circulation electric currents and not by magnetic charges. A comprehensive resource letter by Goldhaber and Trower [16] provides a guide to the literature on magnetic monopoles.

Bridgman underlined the importance of dimensional analysis in his work [17]. Consistent with his idea, many physical problems have been examined using dimensional analysis. Jancewicz [18] used dimensional analysis to find symmetric relations between electromagnetic field quantities and polarizations. However, he did not account for the duality transformation. Artru and Fayolle [19] and then McDonald [20] gave these equations directly. We have used duality and dimensional analysis to construct these equations in SI units.

The Maxwell equations and the Lorentz force [21] equation define the classical dynamics of interacting charged particles and electromagnetic fields. Poynting [22] obtained the electromagnetic energy conservation law

* Department of Electrical and Electronics Engineering, Uludağ University, Bursa, TR-16059, Turkey, omerzor@uludag.edu.tr

DOI: 10.1515/jee-2016-0065, Print (till 2015) ISSN 1335-3632, On-line ISSN 1339-309X 

from the Maxwell equations. The Lorentz force and the Poynting theorem have been defined in many forms in a material medium in the literature. The scalar product of the Lorentz force with the velocity of charges gives the time rate of work conducted by an electromagnetic field on free sources. This time rate of work is equal to the flow of energy plus the time rate of change of stored energy. McDonald [23] noted that Poynting's theorem can be rewritten in many ways with various source/sink terms. McDonald [23] introduced the variations of the Poynting theorem in a material medium without magnetic monopoles. In this work, we give the variations of the Poynting theorem in a material medium with magnetic monopoles.

## 2 FORMULATION

The Maxwell equations define the fields that are radiated by the sources in a medium, but they do not describe the behavior of matter under the influence of the fields.

In a material medium, the presence of electric and magnetic fields influences the motion of bound charges (atomic nuclei and their electrons), inducing local dipole moments. Bound charges are not mobile but elastically bound, contributing, by their slight displacement, to the polarization mechanisms. Purcell [24] proposed the name "structural charges" for bound charges, as these charges are integral parts of the atoms or molecules.

Supplemented material equations need to be defined for a self-consitent solution of the electromagnetic field. We defined these material equations in a linear, non-dispersive, isotropic and homogeneous medium in the presence of magnetic monopoles.

### 2.1 Field quantities in SI units

The microscopic Maxwell equations in the presence of magnetic monopoles,

$$\nabla \cdot \boldsymbol{e}(\boldsymbol{r},t) = \frac{1}{\varepsilon_0}\rho_e(\boldsymbol{r},t)\,, \tag{1}$$

$$\nabla \times \boldsymbol{b}(\boldsymbol{r},t) - \frac{1}{c^2}\frac{\partial \boldsymbol{e}(\boldsymbol{r},t)}{\partial t} = \mu_0 \boldsymbol{j}_e(\boldsymbol{r},t)\,, \tag{2}$$

$$\nabla \cdot \boldsymbol{b}(\boldsymbol{r},t) = \mu_0\rho_m(\boldsymbol{r},t)\,, \tag{3}$$

$$-\nabla \times \boldsymbol{e}(\boldsymbol{r},t) - \frac{\partial \boldsymbol{b}(\boldsymbol{r},t)}{\partial t} = \mu_0 \boldsymbol{j}_m(\boldsymbol{r},t)\,, \tag{4}$$

where $c$ is the speed of light ($c = 1/\sqrt{\varepsilon_0\mu_0}$). We can construct duality transformations from equations (1)–(4),

$$\boldsymbol{e} \to c\boldsymbol{b}\,,\; c\boldsymbol{b} \to -\boldsymbol{e}\,,\; c\rho_e \to \rho_m\,,\; \rho_m \to -c\rho_e\,. \tag{5}$$

In macroscopic electrodynamics, when there are free electric and magnetic charges in a material medium, the electromagnetic field in the medium is radiated by all free and bound charges. Thus, electric and Ampèrian magnetic dipole moments and Gilbertian electric and magnetic dipole moments emerge under the influence of the electromagnetic field. We therefore have two more dipole moments with the existence of magnetic charges. In this part, we determine what volume density of dipole moment needs to be added to which field quantity using dimensional analysis. Hence, we use the subscript "$e$" to show that a vector field is influenced by the motion of electric charges and the subscript "$m$" to show that a vector field is influenced by the motion of magnetic charges.

Electric polarization causes positive and negative charges to gather on opposite sides, either within the material or at its surface along the direction of an applied electric field. If there is a pair of opposite sign, equal magnitude ($q_e$) charges separated by a distance $\ell$, then the electric dipole moment is defined as

$$\boldsymbol{p}_e = q_e\ell\,. \tag{6}$$

The electric polarization vector is a spatial average of the electric dipole moment,

$$\boldsymbol{P}_e = \frac{\boldsymbol{p}_e}{V}\,. \tag{7}$$

As the electric charge is acted upon by the electric and magnetic fields due to both electric and magnetic charges, we can write a constitutive equation using the total macroscopic electric field ($\boldsymbol{E}$)

$$\boldsymbol{D}_e(\boldsymbol{r},t) = \varepsilon_0\boldsymbol{E}(\boldsymbol{r},t) + \boldsymbol{P}_e(\boldsymbol{r},t)\,. \tag{8}$$

Applied electric fields influence the motion of charges in the material. Applied magnetic fields align the axes of the magnetic dipoles formed by the circulating electric charges (electric current) in the material. The magnetic dipole moment due to the fields is the product of the loop current ($I_e$) and the vector area of the loop ($\boldsymbol{A}$),

$$\boldsymbol{m}_e = I_e\boldsymbol{A}\,. \tag{9}$$

The magnetic polarization vector is a spatial average of the magnetic dipole moment,

$$\boldsymbol{M}_e = \frac{\boldsymbol{m}_e}{V}\,. \tag{10}$$

We can write the constitutive equation in the presence of the electric and magnetic charges using the total macroscopic magnetic field ($\boldsymbol{B}$)

$$\boldsymbol{H}_e(\boldsymbol{r},t) = \frac{\boldsymbol{B}(\boldsymbol{r},t)}{\mu_0} - \boldsymbol{M}_e(\boldsymbol{r},t)\,. \tag{11}$$

We can use the duality transformations defined in (5) to obtain the quantities with magnetic monopoles,

$$c\boldsymbol{p}_e \to \boldsymbol{m}_m\,,\; \boldsymbol{m}_e \to -c\boldsymbol{p}_m\,,\; c\boldsymbol{I}_e \to \boldsymbol{I}_m\,. \tag{12}$$

Note that these quantities are corresponded to the electric or magnetic sources ($\rho_e, \rho_m$) for dual symmetry in appropriate manner. If we apply these transformations to the quantities in (6) and (9), we obtain the dual quantities that arise from magnetic monopoles in Table 1.

**Table 1.** Dual quantities in SI units

| | |
|---|---|
| Electric dipole moment $\boldsymbol{p}_e = q_e \ell$ | Gilbertian magnetic dipole moment $\boldsymbol{m}_m = q_m \ell$ |
| Electric polarization $\boldsymbol{P}_e = \boldsymbol{p}_e / V$ | Gilbertian magnetic polarization $\boldsymbol{M}_m = \boldsymbol{m}_m / V$ |
| Ampèrian magnetic dipole moment $\boldsymbol{m}_e = I_e \boldsymbol{A}$ | Gilbertian electric dipole moment $\boldsymbol{p}_m = -\mu_0 \varepsilon_0 I_m \boldsymbol{A}$ |
| Ampèrian magnetic polarization $\boldsymbol{M}_e = \boldsymbol{m}_e / V$ | Gilbertian electric polarization $\boldsymbol{P}_m = \boldsymbol{p}_m / V$ |

The physical dimension of Gilbertian magnetic polarization can be obtained easily in SI units,

$$\left[\boldsymbol{M}_m\right] = \left[\frac{\mathrm{A}}{\mathrm{m}}\right]. \tag{13}$$

Its dimension is the same as the magnetic intensity. Thus, we need not divide this value by $\mu_0$. The Gilbertian magnetic polarization vector can be defined in the same direction as the magnetic field vector,

$$\boldsymbol{H}_m(\boldsymbol{r}, t) = \frac{\boldsymbol{B}(\boldsymbol{r}, t)}{\mu_0} + \boldsymbol{M}_m(\boldsymbol{r}, t) . \tag{14}$$

The physical dimension of the Gilbertian electric polarization can also be obtained easily in SI units,

$$\left[\boldsymbol{P}_m\right] = \left[\frac{\mathrm{C}}{\mathrm{m}^2}\right]. \tag{15}$$

Its dimension is the same as the electric displacement. Therefore, we need not multiply this value by $\varepsilon_0$,

$$\boldsymbol{D}_m(\boldsymbol{r}, t) = \varepsilon_0 \boldsymbol{E}(\boldsymbol{r}, t) - \boldsymbol{P}_m(\boldsymbol{r}, t) . \tag{16}$$

The resulting equations (14) and (16) are, respectively, the dual quantities of (8) and (11) and can help to determine the Maxwell equations in a material medium with magnetic monopoles.

## 2.2 Maxwell equations in material medium with magnetic monopoles

The microscopic Maxwell equations in the presence of magnetic monopoles (1)–(4) are linear and thus allow direct averaging, which reduces to the simple replacement of parameters by their mean values.

In a material medium on a microscopic scale, the bound charge-current densities can be added directly to the free charge-current densities in equations (1)–(4),

$$\rho_e(\boldsymbol{r}, t) \rightarrow \rho_{fe}(\boldsymbol{r}, t) + \rho_{be}(\boldsymbol{r}, t) , \tag{17}$$

$$\boldsymbol{j}_e(\boldsymbol{r}, t) \rightarrow \boldsymbol{j}_{fe}(\boldsymbol{r}, t) + \boldsymbol{j}_{be}(\boldsymbol{r}, t) , \tag{18}$$

$$\rho_m(\boldsymbol{r}, t) \rightarrow \rho_{fm}(\boldsymbol{r}, t) + \rho_{bm}(\boldsymbol{r}, t) , \tag{19}$$

$$\boldsymbol{j}_m(\boldsymbol{r}, t) \rightarrow \boldsymbol{j}_{fm}(\boldsymbol{r}, t) + \boldsymbol{j}_{bm}(\boldsymbol{r}, t) . \tag{20}$$

Here, the known equations of the electric bound charge-current densities are

$$\rho_{be}(\boldsymbol{r}, t) = -\nabla \cdot \boldsymbol{p}_e(\boldsymbol{r}, t) , \tag{21}$$

$$\boldsymbol{j}_{be}(\boldsymbol{r}, t) = \frac{\partial \boldsymbol{p}_e(\boldsymbol{r}, t)}{\partial t} + \nabla \times \boldsymbol{m}_e(\boldsymbol{r}, t) . \tag{22}$$

The magnetic bound charge-current densities can be obtained from equations (21) and (22) using the duality transformations defined in (12),

$$\rho_{bm}(\boldsymbol{r}, t) = -\nabla \cdot \boldsymbol{m}_m(\boldsymbol{r}, t) , \tag{23}$$

$$\boldsymbol{j}_{bm}(\boldsymbol{r}, t) = \frac{\partial \boldsymbol{m}_m(\boldsymbol{r}, t)}{\partial t} - \frac{1}{\mu_0 \varepsilon_0} \nabla \times \boldsymbol{p}_m(\boldsymbol{r}, t) . \tag{24}$$

By applying a statistical average over a smooth region, the averaged field and source quantities can be obtained. Thus, the microscopic field and source quantities in the linear Maxwell equations can be replaced directly by macroscopic field and source quantities.

$$\begin{aligned}\nabla \times \boldsymbol{E}(\boldsymbol{r}, t) + \frac{\partial \boldsymbol{B}(\boldsymbol{r}, t)}{\partial t} = -\mu_0 \Big(& \boldsymbol{J}_{fm}(\boldsymbol{r}, t) + \\ & \frac{\partial \boldsymbol{M}_m(\boldsymbol{r}, t)}{\partial t} - \frac{1}{\mu_0 \varepsilon_0} \nabla \times \boldsymbol{P}_m(\boldsymbol{r}, t) \Big),\end{aligned} \tag{25}$$

$$\begin{aligned}&\nabla \times \boldsymbol{B}(\boldsymbol{r}, t) - \frac{1}{c^2} \frac{\partial \boldsymbol{E}(\boldsymbol{r}, t)}{\partial t} = \\ &\quad \mu_0 \left( \boldsymbol{J}_{fe}(\boldsymbol{r}, t) + \frac{\partial \boldsymbol{P}_e(\boldsymbol{r}, t)}{\partial t} + \nabla \times \boldsymbol{M}_e(\boldsymbol{r}, t) \right),\end{aligned} \tag{26}$$

$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r}, t) = \frac{1}{\varepsilon_0} (\bar{\rho}_{fe}(\boldsymbol{r}, t) - \nabla \cdot \boldsymbol{P}_e(\boldsymbol{r}, t)), \tag{27}$$

$$\nabla \cdot \boldsymbol{B}(\boldsymbol{r}, t) = \mu_0 (\bar{\rho}_{fm}(\boldsymbol{r}, t) - \nabla \cdot \boldsymbol{M}_m(\boldsymbol{r}, t)). \tag{28}$$

If we use the constitutive equations in (8), (11), (14) and (16), we obtain a macroscopic Maxwell equation in a form in which only free sources appear (similar forms were given directly by Artru and Fayolle [19] and McDonald [20]),

$$-c^2 \nabla \times \boldsymbol{D}_m(\boldsymbol{r}, t) - \frac{\partial \boldsymbol{H}_m(\boldsymbol{r}, t)}{\partial t} = \boldsymbol{J}_{fm}(\boldsymbol{r}, t) , \tag{29}$$

$$\nabla \times \boldsymbol{H}_e(\boldsymbol{r}, t) - \frac{\partial \boldsymbol{D}_e(\boldsymbol{r}, t)}{\partial t} = \boldsymbol{J}_{fe}(\boldsymbol{r}, t) , \tag{30}$$

$$\nabla \cdot \boldsymbol{D}_e(\boldsymbol{r}, t) = \bar{\rho}_{fe}(\boldsymbol{r}, t) , \tag{31}$$

$$\nabla \cdot \boldsymbol{H}_m(\boldsymbol{r}, t) = \bar{\rho}_{fm}(\boldsymbol{r}, t) . \tag{32}$$

### 2.3 The Poynting theorem in material medium with magnetic monopoles

An electric field ($\boldsymbol{e}$) in microscopic electrodynamics is proportional to the electric displacement vector ($\boldsymbol{d}$), with the multiplicative constant ($\epsilon_0$) depending on the physical units. Similarly, a magnetic field ($\boldsymbol{b}$) in microscopic electrodynamics is proportional to the magnetic intensity ($\boldsymbol{h}$), with the multiplicative constant ($\mu_0$) depending on the physical units.

In macroscopic electrodynamics, electric polarization and Ampèrian magnetization charges and currents emerge inside an Ampèrian magnetic material illuminated by an electromagnetic field. Thus, if a magnetic charge moves along a closed path, any part of which passes through an Ampèrian magnetic material, then the energy could be extracted from the system. Gilbertian electric and magnetic polarization charges and currents emerge inside a Gilbertian magnetic material illuminated by an electromagnetic field. Thus, if an electric charge moves along a closed path, any part of which passes through a Gilbertian magnetic material, then energy could be extracted from the system. Thus, the internal dynamics of the material medium changes the equilibrium of the energy of the moving charges, with the effect of external fields. In accordance with this proposition, the Lorentz force and the Poynting theorem have new definitions in the material medium.

First, we introduce Lorentz force definitions for the electric charge and magnetic monopole. In microscopic electrodynamics, an electromagnetic force acts on a moving electric charge with velocity $\boldsymbol{v}$ as

$$\boldsymbol{f}_e(\boldsymbol{r},t) = q_e(\boldsymbol{e}(\boldsymbol{r},t) + \mathbf{v}(t) \times \boldsymbol{b}(\boldsymbol{r},t))\,. \tag{33}$$

The electromagnetic force on a moving magnetic monopole with velocity $\boldsymbol{v}$ can be obtained using the duality transformations in (5) on (33),

$$\boldsymbol{f}_m(\boldsymbol{r},t) = q_m(\boldsymbol{b}(\boldsymbol{r},t) - \frac{\mathbf{v}(t)}{c^2} \times \boldsymbol{e}(\boldsymbol{r},t))\,. \tag{34}$$

In macroscopic electrodynamics, the force on free charges and current densities can be defined in two sets of equations, which have been proposed by various scientists (in the second set, our new field quantities are used),

$$\boldsymbol{F}_e(\boldsymbol{r},t) = \bar{\rho}_{fe}(\boldsymbol{r},t)\boldsymbol{E}(\boldsymbol{r},t) + \boldsymbol{J}_{fe}(\boldsymbol{r},t) \times \boldsymbol{B}(\boldsymbol{r},t)\,, \tag{35}$$

$$\boldsymbol{F}_m(\boldsymbol{r},t) = \bar{\rho}_{fm}(\boldsymbol{r},t)\boldsymbol{B}(\boldsymbol{r},t) - \frac{\boldsymbol{J}_{fm}(\boldsymbol{r},t)}{c^2} \times \boldsymbol{E}(\boldsymbol{r},t)\,, \tag{36}$$

or

$$\boldsymbol{F}_e(\boldsymbol{r},t) = \frac{1}{\epsilon_0}\bar{\rho}_{fe}(\boldsymbol{r},t)\boldsymbol{D}_m(\boldsymbol{r},t) + \mu_0\boldsymbol{J}_{fe}(\boldsymbol{r},t) \times \boldsymbol{H}_m(\boldsymbol{r},t)\,, \tag{37}$$

$$\boldsymbol{F}_m(\boldsymbol{r},t) = \mu_0(\bar{\rho}_{fm}(\boldsymbol{r},t)\boldsymbol{H}_e(\boldsymbol{r},t) - \boldsymbol{J}_{fm}(\boldsymbol{r},t) \times \boldsymbol{D}_e(\boldsymbol{r},t))\,. \tag{38}$$

We analyzed the electromagnetic force on moving charges. The rate of work on electric charges and magnetic charges in microscopic and macroscopic electrodynamics, containing Lorentz force vectors, are given respectively,

$$\boldsymbol{f}_e(\boldsymbol{r},t) \cdot \mathbf{v}(t)\;, \quad \boldsymbol{f}_m(\boldsymbol{r},t) \cdot \mathbf{v}(t), \tag{39}$$

$$\boldsymbol{F}_e(\boldsymbol{r},t) \cdot \mathbf{v}(t)\;, \quad \boldsymbol{F}_m(\boldsymbol{r},t) \cdot \mathbf{v}(t). \tag{40}$$

The rate of work conducted by electromagnetic fields on free charges and monopoles in macroscopic electrodynamics can be obtained using (37) and (38),

$$\begin{aligned}\frac{d\omega(\boldsymbol{r},t)}{dt} &= \boldsymbol{F}_e(\boldsymbol{r},t) \cdot \boldsymbol{v}(t) + \boldsymbol{F}_m(\boldsymbol{r},t) \cdot \boldsymbol{v}(t) = \\ &\frac{1}{\epsilon_0}\boldsymbol{J}_{fe}(\boldsymbol{r},t) \cdot \boldsymbol{D}_m(\boldsymbol{r},t) + \mu_0\boldsymbol{J}_{fm}(\boldsymbol{r},t) \cdot \boldsymbol{H}_e(\boldsymbol{r},t)\,.\end{aligned} \tag{41}$$

We use Maxwell equations (29) and (30) to obtain this equation with field quantities. We take the scalar product of (29) with $\mu_0\boldsymbol{H}_e$ on both sides, and then we take the scalar product of (30) with $1/\epsilon_0\boldsymbol{D}_m$ on both sides. By summing these equations, we obtain the Poynting theorem with field quantities,

$$\begin{aligned}\frac{d\omega(\boldsymbol{r},t)}{dt} &= -\frac{1}{\epsilon_0}\nabla \cdot (\boldsymbol{D}_m(\boldsymbol{r},t) \times \boldsymbol{H}_e(\boldsymbol{r},t)) \\ &- \frac{1}{\epsilon_0}\boldsymbol{D}_m(\boldsymbol{r},t) \cdot \frac{\partial \boldsymbol{D}_e(\boldsymbol{r},t)}{\partial t} - \mu_0\boldsymbol{H}_e(\boldsymbol{r},t) \cdot \frac{\partial \boldsymbol{H}_m(\boldsymbol{r},t)}{\partial t} \\ &\equiv -\left(\nabla \cdot \boldsymbol{S}(\boldsymbol{r},t) + \frac{\partial u(\boldsymbol{r},t)}{\partial t}\right).\end{aligned} \tag{42}$$

The Poynting vector in any medium is

$$\boldsymbol{S}(\boldsymbol{r},t) = \frac{1}{\epsilon_0}\boldsymbol{D}_m(\boldsymbol{r},t) \times \boldsymbol{H}_e(\boldsymbol{r},t)\,. \tag{43}$$

The total density stored in an electromagnetic field in linear medium is

$$u(\boldsymbol{r},t) = \frac{1}{2\epsilon_0}\boldsymbol{D}_e(\boldsymbol{r},t) \cdot \boldsymbol{D}_m(\boldsymbol{r},t) + \frac{\mu_0}{2}\boldsymbol{H}_e(\boldsymbol{r},t) \cdot \boldsymbol{H}_m(\boldsymbol{r},t)\,. \tag{44}$$

### 2.4 Variants of Poynting theorem

As noted by McDonald [23], the Poynting theorem generally defines the flow of electromagnetic energy and does not definitely identify the electromagnetic energy density. He proposed that many definitions can be constructed using electrical and magnetic field quantities. McDonald [23] introduced the idea that, in a material medium without magnetic monopoles, the Poynting theorem could be defined in $3^6 = 729$ variants with the help of permutation using the equalities of field quantities ($\boldsymbol{D} = \epsilon_0\boldsymbol{E} + \boldsymbol{P}$), ($\boldsymbol{H} = \boldsymbol{B}/\mu_0 - \boldsymbol{M}$).

In this work, we calculate the variants of the Poynting theorem in a material medium with magnetic monopoles. We can permutate the field quantities in equations (8), (11), (14) and (16) through the Poynting theorem in (42) to obtain $5^6 = 15625$ variants. One of them that is different from (42) is given below,

$$\begin{aligned}\frac{d\omega(\boldsymbol{r},t)}{dt} &= -\frac{1}{\epsilon_0}\nabla\cdot(\boldsymbol{D}_e(\boldsymbol{r},t)\times\boldsymbol{H}_m(\boldsymbol{r},t)) \\ &\quad -\frac{1}{\epsilon_0}\boldsymbol{D}_e(\boldsymbol{r},t)\cdot\frac{\partial\boldsymbol{P}_e(\boldsymbol{r},t)}{\partial t} - \mu_0\boldsymbol{M}_m(\boldsymbol{r},t)\cdot\frac{\partial\boldsymbol{H}_e(\boldsymbol{r},t)}{\partial t} \\ &\equiv -\left(\nabla\cdot\boldsymbol{S}(\boldsymbol{r},t) + \frac{\partial u(\boldsymbol{r},t)}{\partial t}\right). \qquad (45)\end{aligned}$$

These variations consist of the standard Poynting theorem and Poynting theorems in which the source terms are electromagnetic.

## 3 CONCLUSION

We have assumed the existence of magnetic monopoles in material medium and, in this manner, defined polarization vectors of magnetic monopoles using the duality transformation. Then, the displacement and magnetic intensity vectors (constitutive equations) were described in SI units using dimensional analysis. Finally, symmetric Maxwell equations with new field quantities, in which only free sources appear, were introduced. Thus, the Lorentz force and the Poynting theorem were defined using these new field quantities, and many possible definitions of them were given.

As magnetic monopoles have not been observed in the real world, it is difficult to define certain physical quantities including magnetic monopoles. Therefore, duality and dimensional analysis are very helpful instruments in describing these quantities.

### Acknowledgement

Thanks to Kirk T. McDonald for his useful comments which serve as support.

## References

[1] MAXWELL, J. C.: A Treatise on Electricity and Magnetism, 3rd ed., vol. 1, Oxford University Press, New York, 2002.

[2] HEAVISIDE, G.: Some Properties of Maxwell Equations, Phil. Trans. Roy. Soc. A **183** (1893), 423–430.

[3] LARMOR, I.: Collected Papers, London, 1928.

[4] McDONALD, K. T.: Electrodynamics in 1 and 2 Spatial Dimensions (October 16, 2014), http://princeton.edu/∼mcdonald/examples/2dem.pdf.

[5] POINCARÉ, H.: Remarques sur une Expérience de M. Birkeland, Compt. Rend. **123** (1896), 530–533.

[6] THOMSON, J. J.: Electricity and Matter, Scribners, New York, 1904.

[7] DIRAC, P. A. M.: Quantised Singularities in the Electromagnetic Field, Proc. Roy. Soc. Lond. **A133** (1931), 60–72.

[8] DIRAC, P. A. M.: The Theory of Magnetic Monopoles, Phys. Rev. **74** (1948), 817-830.

[9] SCHWINGER, J.: Magnetic Charge and Quantum Field Theory, Phys. Rev. **144** (1966), 1087–1093.

[10] SCHWINGER, J.: Sources and Magnetic Charge, Phys. Rev. **173** (1968), 1536–1544.

[11] SCHWINGER, J.: Magnetic Charge and the Charge Quantization Condition, Phys. Rev. D **12** (1975), 3105–3111.

[12] SCHWINGER, J.: A Magnetic Model of Matter, Science **165** (1969), 757–761.

[13] ZWANZIGER, D.: Local-Lagrangian Quantum Field Theory of Electric and Magnetic Charges, Phys. Rev. D **3** (1971), 880–891.

[14] WU, T. T.—YANG, C. N.: Concept of Nonintegrable Phase Factors and Global Formulation of Gauge Fields, Phys. Rev. D **12** (1975), 3845–3857.

[15] JACKSON, J. D.: The Nature of Intrinsic Magnetic Dipole Moments, CERN-77–17, 1977.

[16] GOLDHABER, A. S.—TROWER, W. P.: Resource Letter MM-1: Magnetic monopoles, Am. J. Phys. **58** (1990), 429–439.

[17] BRIDGMAN, P. W.: Dimensional Analysis, in Encyclopaedia Britannica, Wm. Haley, Editor-in-Chief, vol. 7, Chicago, 1969, pp. 439–449.

[18] JANCEWICZ, B.: Electromagnetic Polarizations, Bulg. J. Phys. **29** (2002), 151–154.

[19] ARTRU, X.—FAYOLLE, D.: Dynamics of a Magnetic Monopole in Matter, Maxwell Equations in Dyonic Matter and Detection of Electric Dipole Moments, Prob. Atom. Sci. Tech. **6** (2001).

[20] McDONALD, K. T.: Poynting's Theorem with Magnetic Monopoles, (May 1, 2013), http://princeton.edu/∼mcdonald/examples/poynting.pdf.

[21] HEAVISIDE, O.: On the Electromagnetic Effects due to the Motion of Electrification through a Dielectric, Phil. Mag. **27** No. 324 (1889).

[22] POYNTING, J. H.: On the Transfer of Energy in the Electromagnetic Field, Phil. Trans. Roy. Soc. London **175** No. 343 (1884).

[23] McDONALD, K. T.: Variants of Poynting's Theorem (May 21, 2013), http://www.hep.princeton.edu/∼mcdonald/examples/variants.pdf.

[24] PURCELL, E. M.—MORIN, D. J.: Electricity and Magnetism, 3rd ed., Cambrige University Press, 2013.

Received 24 March 2016

**Ömer Zor** received BSc, MSc and PhD in Electronics Engineering from Uludağ University. He worked as a research assistant at Mathematics and Electronics Engineering Departments of Uludağ University between 2006-2013. Currently he works at Electrical and Electronics Engineering Department of the same University. His research interests are on electrodynamics and analytical methods in electromagnetic theory.