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A NUMERICAL INVESTIGATION ON THE STRESS-BALANCED SPUR GEAR PAIRS

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Abstract: Involute spur gears are key machine elements for power transmission in various industrial sectors. During power transmission, the teeth are subjected to high stresses. Many design modifications are used to reduce these stresses, such as increasing the drive side pressure angle and profile shifting factor. These modifications change the contact ratio and center distance of the gear pair. Changing the tooth thickness is another solution to reduce stress. Because there is a difference in the number of teeth between the pinion and the gear, the pinion stress levels are higher than the gear for the same gear parameters. The tooth thickness value in the tooth thickness on the pitch circle is equal to 0.5xπm for both pinion and gear as standard. Stress compensation can be achieved by increasing this thickness at the pinion and decreasing it at the same rate at the gear. In this study, first of all, 3D designs of gears with non-standard thickness were created in CATIA and finite element analyzes were performed to obtain tooth thickness values that create equal root stresses for pinion and gear with various tooth numbers. According to preliminary results, tooth deformation and stress has a linear relationship with tooth thickness value, nearly.

Keywords: 3D design of gear, Finite element analysis, Tooth thickness

Dengeli Gerilmeye Sahip Dişli Çark Çiftleri Üzerine Nümerik bir Araştırma

Öz: Evolvent düz dişliler, çeşitli endüstriyel sektörlerde güç aktarımı için kilit makine elemanlarıdır. Güç aktarma esnasında dişler yüksek gerilmelere maruz kalır. Bu gerilmeleri azaltmak için, süren taraf kavrama açısını ve profil kaydırma faktörünü arttırma gibi birçok tasarım modifikasyonu kullanılmaktadır. Bu modifikasyonlar, dişli çiftinin kavrama oranını ve eksenler arası mesafesini değiştirir. Diş kalınlığını değiştirmek, gerilmeyi azaltmak için başka bir çözümdür. Pinyon ve dişli arasında diş sayısı farkı olduğundan, aynı dişli parametreleri için pinyon gerilme düzeyleri dişliden daha yüksektir. Taksimat dairesindeki diş kalınlığı değeri standart olarak hem pinyon hem de dişli için 0,5xπm'ye eşittir. Bu kalınlığı pinyonda artırmak ve aynı oranda dişlide azaltmak suretiyle gerilme dengelemesi yapılabilir. Bu çalışmada öncelikle standart olmayan kalınlığa sahip dişlilerin 3 boyutlu tasarımları CATIA da oluşturulmuş ve sonlu eleman analizleri yapılarak çeşitli diş sayıları ile pinyon ve dişli için eşit diş dibi gerilmeleri oluşturan diş kalınlık değerleri elde edilmiştir. Ön sonuçlara göre, diş gerilme ve deformasyonu, diş kalınlığı değeri ile hemen hemen doğrusal bir ilişkiye sahiptir.

Anahtar Kelimeler: 3B Dişli tasarımı, Sonlu elemanlar yöntemi, Diş kalınlığı

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1. INTRODUCTION

Recently, decreasing resources of oil and rising risk of global warming are lead to investigate ways of energy-consuming reduction to reduce CO2 emission levels and air pollution for sustainability. It is expected that energy demand in the world will increase by nearly 50% in the next 15 years (Yuce et al., 2014). In-vehicle industry, there are several methods to increase energy efficiency to ensure this aim. Among them, reducing the total weight of vehicles comes forward overwhelmingly since approximately 75% of fuel usage is directly related to it (Yilmaz et al., 2017).

Re-designing components of the vehicle are one of the alternative ways of weight reduction. Spur gears are a popular power transmissin element in the vehicle industry. As standard, meshing gears have the same tooth thickness on the pitch circle ($tp=tg=0.5\times\pi m$). Actually, it is not a mandatory situation. Meshing gears could be designed with unequal tooth thickness. With this change, the stress values on pinion and gear could be equal. It leads to increase load-carrying capacity, and it may reduce the weight of the gear mechanism.

Design and stress analysis of standard and nonstandard spur gears have been investigated for many years (Litvin, 2004 and Colbourne, 1987). Using asymmetric gear is one the sample of stress reduction of gears. FEA study is conducted to investigate the effects of pressure angle on stress levels in the study. It is found that using a larger pressure angle on the drive side decreases stress levels, so it may result in using low-sized gears (Cavdar et al., 2005). Another example is using an extra larger tool tip radius for generating the drive side of gear. This study, it is analyzed stress reduction with increasing tip radius via FEM. According to the results, using an extra larger tip radius leads to decrease maximum bending stresses (Yilmaz et al., 2017, Dogan et al., 2018). Using unequal tooth thickness to reduce and balance stress levels is a relatively novel method. It is proposed to use different tooth thicknesses on pinion and gear in this study. FEA is conducted for different gear parameters (Sekar et al., 2015).In this study, the effects of using unequal tooth thickness are investigated on wear. With increasing tooth thickness, it is reached that the wear strength of the pinion increases (Sekar et al., 2017).The influence of changing tooth thickness on power loss and efficiency is examined in this study. It is found that using larger tooth thickness on the pinion results in decreasing power loss (Sekar et al., 2015).

In this study, first, for the design of the nonstandard gear, a MATLAB code is prepared. The points of nonstandard gear tooth are exported to CATIA. 2D CAD Model is used for finite element analyses in ANSYS. The specific tooth thickness value is obtained for the conducted case study. For specific tooth thickness, the tooth deformation values are obtained. After that, the changing weight value is determined.

2. MATERIALS AND METHOD

In this section, the mathematical expression of spur gear is presented using rack cutter mathematical equations, differential geometry, and gearing theory. The parameters of the rack cutter are illustrated in Figure 1. The coordinate system S_n (X_n,Y_n) is located in the middle of pitch line on tooth space (Litvin, 2004).

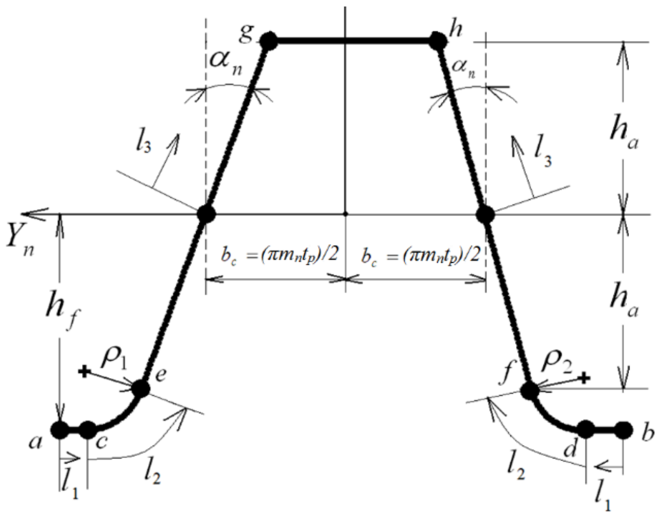


Figure 1:
Details of rack cutter

The matrixes of portions in Figure 1 are given in the following equations (Yilmaz et al., 2017).

ac-bd region

$$R_n^1=\begin{bmatrix} -h_f \\ \pm(\frac{\pi m}{2}-l_1) \\ 0 \\ 1 \end{bmatrix} \tag{1}$$

$$0<l_1<b_c-h_f\tan\alpha_n+\rho_{1,2}\tan\alpha_n-\rho_{1,2}\sec\alpha_n \tag{2}$$

ce-df region

$$R_n^2=\begin{bmatrix} -h_f+\rho_{1,2}-\rho_{1,2}\cos l_2 \\ \pm(b_c+h_f\tan\alpha_n-\rho_{1,2}\tan\alpha_n+\rho_{1,2}\sec\alpha_n-\rho_{1,2}\sin(l_2)) \\ 0 \\ 1 \end{bmatrix} \tag{3}$$

$$0<l_2<(\frac{\pi}{2}-\alpha_n) \tag{4}$$

eg-fh region

$$R_n^3=\begin{bmatrix} l_3\cos\alpha_n \\ \pm(b_c-l_3\sin\alpha_n) \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

$$\frac{-h_a}{\cos\alpha_n} \leq l_3 \leq \frac{h_a}{\cos\alpha_n} \tag{6}$$

Where, m is the module, z is the teeth number, $\alpha_{n1,2}$ is the pressure angle on sides, h_f is the dedendum, h_a is the addendum, $\rho_{1,2}$ are the tip radii, $l_{1,2,3}$ is the design parameter of the cutter, b_c is half the thickness of rack on pitch line.

To obtain the roll angle for each region unit normal vectors are presented in the following equation (Yilmaz et al., 2017).

$$n_n^i = \frac{\frac{\partial R_n^i}{\partial l_i} x k_n}{\left| \frac{\partial R_n^i}{\partial l_i} x k_n \right|} \qquad i=1\text{-}3 \tag{7}$$

Where k_n unit normal vector of Z direction.

According to the gearing theory, direction of the sliding velocity vector between pinion and gear is parallel with the tangent vector of the common meshing point. Of course, it is always perpendicular to a common normal vector. This expression is presented in Eq. (8) (Yilmaz et al., 2017).

$$n_n^i . v_{relative} = 0 \qquad i=1\text{-}3 \tag{8}$$

During the generating process, the rack cutter makes a linear motion as $r_{p1} \times \phi_1$ whilst the gear as a workpiece revolves as $Ø_1$. $S_1(X_1, Y_1)$ is the coordinate system of the workpiece. The relationship between cutter and workpiece is shown in Figure 2 (Dogan et al., 2018).

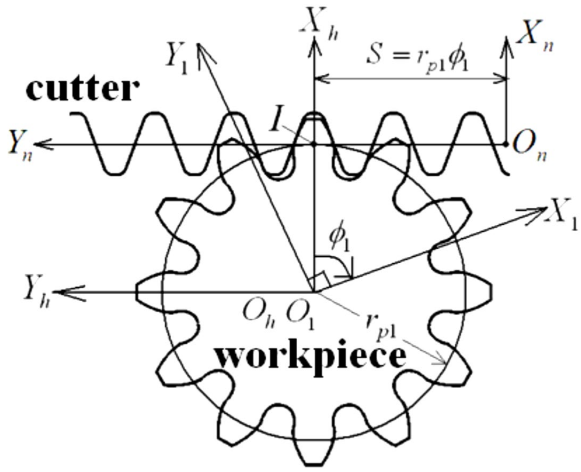


Figure 2:
Relation of cutter and workpiece

The coordinate transformation matrix between rack cutter and workpiece is presented in the following equations [6].

$$M_{1n}=\begin{bmatrix} \cos(\emptyset_1) & -\sin(\emptyset_1) & 0 & r_{p1}\emptyset_1\sin(\emptyset_1)+r_{p1}\cos(\emptyset_1) \\ \sin(\emptyset_1) & \cos(\emptyset_1) & 0 & -r_{p1}\emptyset_1\cos(\emptyset_1)+r_{p1}\sin(\emptyset_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

$$R_1^i=M_{1n}{}^iR_n{}^i \qquad i=1\text{-}3 \tag{10}$$

Where M_{1n} is the coordinate transformation matrix and, R_1 is the matrix of involute spur gear, r_{p1} is pitch diameter. With programming Eq.(1-10) in the MATLAB program, the design points of involute spur gears are obtained. These points are exported to the CATIA program to generate the FE model. In Figure 3, sections of design phases are illustrated [6].

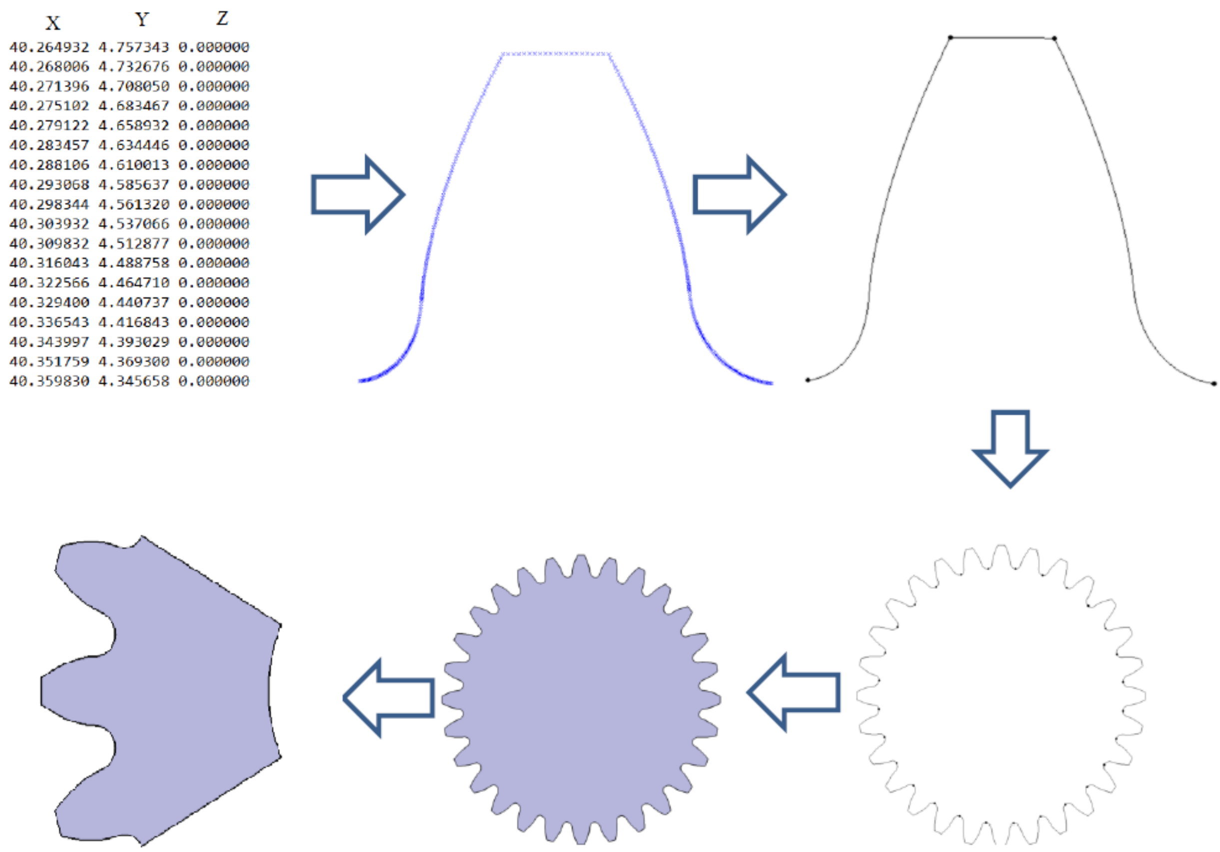


Figure 3:
Design phases of spur gears

3. FINITE ELEMENT ANALYSES

In this section, finite element analyses are conducted for predefined pinion and gear parameters. ANSYS Workbench is used for this aim. The effect of tooth thickness on the pitch circle on bending stress is investigated in a case study. The values of tooth thickness that make stresses equal to both pinion and gear are determined. In Table 1, the parameter of the case study is illustrated.

Table 1: Gear parameters

Parameters	*Pinion*	*Gear*
Module-m (mm)	4	4
Teeth number(z)	20	40
Pressure angle-α_n (°)	20	20
Addendum-h_a (×m)	1	1
Dedendum-h_f (×m)	1.25	1.25
Cutter tip radius- $\rho_{1,2}$ (×m)	Fully rounded	Fully rounded
Facewidth- b (mm)	20	20
Rim thickness- S_R (×m)	5	5
Profile shifting- x (mm)	0	0
Tooth thickness on the pitch circle(t_p) (×πm)	0.5-0.52-0.54-0.56-0.58	0.5-0.48-0.46-0.44-0.42

Three teeth model is prepared for finite element analysis. Meshing force (80 N) is applied at the highest point of single tooth contact (HPSTC). Fixed support is given to lateral sides and shaft hole. Boundary conditions and mesh structure are presented in Figure 4.

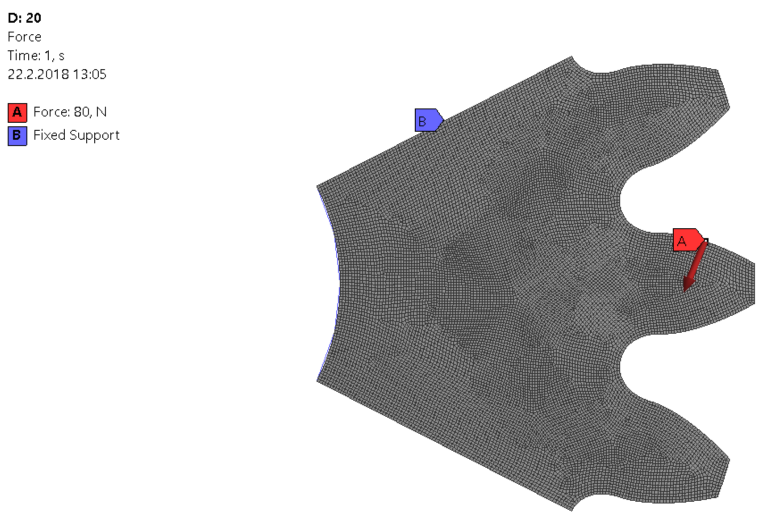
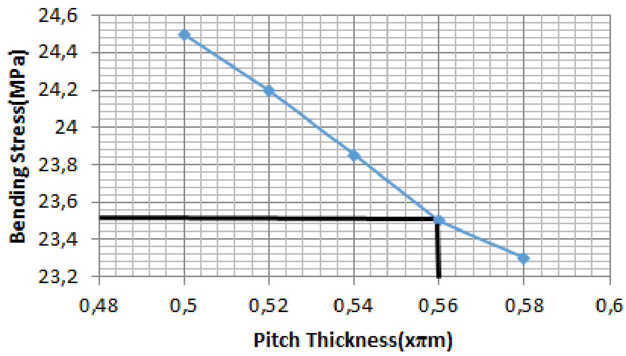
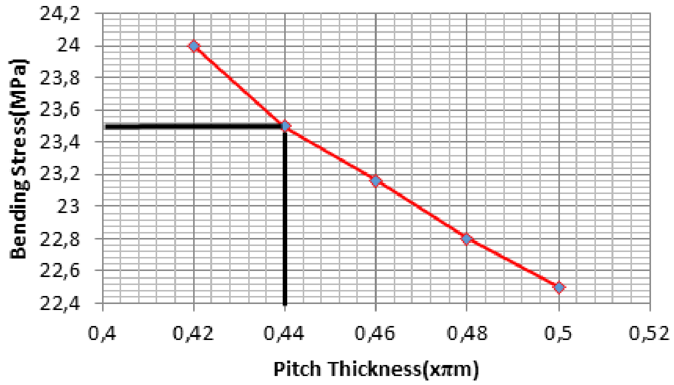


Figure 4:
Load and boundary conditions

Since the low facewidth to tooth height ratio, 2D plane stress theory is used for analysis. A quadrilateral (Plane 82) mesh element with a 0.05 mesh size is used. In Figure 5, the effects of tooth thickness on the pitch circle on gear root stress for pinion and gear are illustrated.



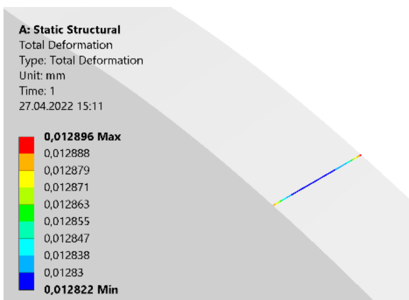
a) Pinion



b) Gear

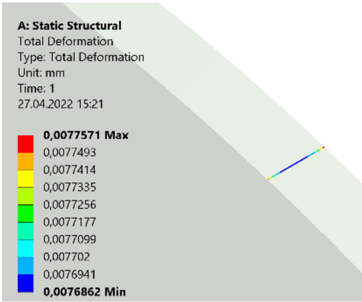
Figure 5:
Root stress results

It is clearly seen that with increasing tooth thickness on the pitch circle, the maximum bending stress decreases for the selected case study when pinion with 0.56×πm tooth thickness on the pitch circle and gear with 0.44×πm tooth thickness on the pitch circle, the maximum bending stress reached a balance value which is approximately 23.5 MPa. In Figure 6, the tooth deformation values are illustrated for predetermined tooth thickness values. In deformation analyses, force is applied to tooth thickness on the pitch circle line.

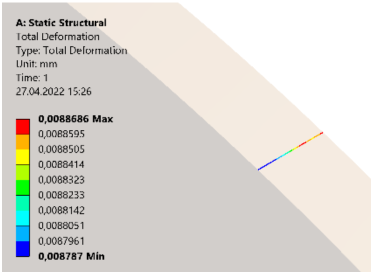


a) z=20, 0.5 xπm tooth thickness

b) z=20, 0.56 xπm tooth thickness



c) z=40, 0.5 xπm tooth thickness



d) z=40, 0.44xπm tooth thickness

Figure 6:
Tooth deformation results on the pitch circle line

Changing the tooth thickness of gear also affects the tooth deformation. Tooth deformation is another important parameter as it directly determines the tooth stiffness value. After specifying the tooth thickness on the pitch circle value, which makes equal the pinion and gear root stress, the tooth deformation values on the pitch line of the involute flank are also obtained with finite element analysis. According to the results, when the tooth thickness is increased from 0.5x πm to 0.56x πm, the tooth deformation value decreases by 13.5%, nearly for z=20. For z=40, tooth deformation increases 12.5% approximately when the tooth thickness value decreases from 0.5xπm to 0.44xπm.

Another result of this situation, “the weight reduction” for obtained tooth thickness on the pitch circle value is illustrated in Table 2.

Table 2: Weight status of gears

Tooth thickness on the pitch circle (t_p)	*Pinion weight(gr)*	*Gear weight(gr)*
0,5×πm	657.724	1643.802
0,56×πm	681.44	-
0,44×πm	-	1597.696
Difference	+23.726	-46.106
Total	-22.38	

4. CONCLUSION

In this study, the effect of tooth thickness on the pitch circle on gear stress is investigated. For this, first, involute spur gear with nonstandard tooth thickness on the pitch circle is designed with programming equations nonstandard rack cutter. Then for finite element analysis, three teeth model is prepared in the CAD program. In the ANSYS package, the analyses are conducted. For selected parameters, the specific tooth thicknesses values which make bending stresses equal for both pinion and gear are found via these analyses. According to the results, for stress balancing without changing other parameters, it is required for pinion, the tooth thickness value has to be increased, and for gear, its value has to be decreased to ensure total pitch value(πm). After that, the effect of tooth thickness on the tooth deformation value is investigated for specific pitch tooth thickness values. Root stress and deformation increases by 5% and 12.5% for gear after the design changes. For pinion, root stress decreases by 4% while deformation decreases 13.5%. Lastly, changing the weight of gears is determined via the CAD program. Approximately 22 gr weight reduction is obtained. When taking into account the number of vehicles and gear pairs in a vehicle, this reduction ensures substantial savings.

CONFLICT OF INTEREST

Author(s) approve that to the best of their knowledge, there is not any conflict of interest or common interest with an institution/organization or a person that may affect the review process of the paper.

AUTHOR CONTRIBUTION

Tufan Gürkan Yılmaz is responsible for determining the concept and design process of the research, Gültekin Karadere is responsible for finite element analyses, Fatih Karpat is responsible for data analysis and interpretation of results.

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