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Conservation laws for perturbed solitons in optical metamaterials

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ABSTRACT

The conservation laws for the dynamics of soliton propagation through optical metamaterials are derived by the aid of Lie symmetry analysis. The proposed model will be studied with two forms of nonlinearity. They are Kerr law and parabolic law.

Introduction

Performance enhancement is one of the key issues in the study of soliton propagation through optical waveguides. In this context, there are various kinds of waveguides that has been lately considered [1–10]. A few of them are optical fibers, couplers, PCF, optical metamaterials and metasurfaces. This paper focuses on a particular issue in the soliton propagation dynamics through optical metamaterials. It is the aspect of conservation laws. No study of a physical phenomena is complete without a listing of conserved quantities. While there are several papers visible in this area, it was never reported earlier about conserved quantities for soliton propagation through such metamaterials. This paper thus fills in the gap. The conservation laws for optical metamaterials are presented when a few Hamiltonian type perturbation terms are taken into account. There are two types of nonlinearity that successfully retrieves these conserved densities by the aid of Lie symmetry analysis. These are Kerr law and parabolic law. The conserved quantities are subsequently computed from bright 1-soliton solutions that were reported earlier. These are detailed in the following sections.

Governing equation

The dynamics of solitons in optical metamaterials is governed by the nonlinear Schrödinger's equation (NLSE) which in the dimensionless form is given by [2,10]

$$iq_t + aq_{xx} + F\left(|q|^2\right)q = i\alpha q_x + i\lambda\left(|q|^2 q\right)_x + i\nu\left(|q|^2\right)_x q + \theta_1\left(|q|^2 q\right)_{xx} + \theta_2|q|^2 q_{xx} + \theta_3 q^2 q^*_{xx} \tag{1}$$

Eq. (1) is the NLSE that is studied in the context of metamaterials. Here in (1), a and b are the group velocity dispersion and the self-phase modulation terms respectively. This pair produces the delicate balance between dispersion and nonlinearity that accounts for the formation of the stable solitons.On the right-hand side λ represents the self-steepening term in order to avoid the formation of shocks and ν is the nonlinear dispersion,while α represents the inter-modal dispersion. This arises from the fact that group velocity of light in multi-mode fibers depends on chromatic dispersion as well as the propagation mode involved. Next, θ_j for $j = 1, 2, 3$ are the perturbation terms that appear in the context of metamaterials. Finally, the independent variables are x and t that represent spatial and temporal variables respectively with the dependent variable $q(x, t)$ being the complex-valued wave profile.

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The real-valued algebraic functional F must possess smoothness of the complex-valued function $F\left(|q|^2\right)q : C \to C$. Treating the complex plane C as two-dimensional linear space R^2, the function $F\left(|q|^2\right)q$ is k times continuously differentiable provided

$$F\left(|q|^2\right)q \in \bigcup_{m,n=1}^{\infty} C^k((-n,n)\ \times\ (-m,m);\ R^2). \tag{2}$$

Conserved densities

In order to determine conserved densities and fluxes, we resort to the invariance and multiplier approach based on the well known result that the Euler-Lagrange operator annihilates a total divergence [5]. Firstly, if (T^t, T^x) is a conserved vector corresponding to a conservation law, then

$$D_tT^t + D_xT^x = 0 \tag{3}$$

along the solutions of the differential equation (de = 0). For a scalar pde $E(x,t,q,q_{(1)},\ldots) = 0$, if there exists a nontrivial differential function Q, called a 'multiplier', such that

$$E_q[QE] = 0$$

then QE is a total divergence, i.e.,

$$QE = D_tT^t + D_xT^x$$

for some (conserved) vector (T^t, T^x) – E_q is the respective Euler–Lagrange operator. Thus, a knowledge of each multiplier Q leads to a conserved vector determined by, amongst other things, a Homotopy operator.

$$E^1(x,t,u,v,u_{(1)},v_{(1)},\ldots) = 0, \quad E^2(x,t,u,v,u_{(1)},v_{(1)},\ldots) = 0 \tag{4}$$

For a system (4), $Q = (Q^1, Q^2)$, say, so that

$$Q^1\left(E^1\right) + Q^2\left(E^2\right) = D_tT^t + D_xT^x$$

and

$$E_{(u,v)}\left[D_tT^t + D_xT^x\right] = 0.$$

In each case, T^t is the conserved density.

Kerr law

This law is also known as the cubic nonlinearity and is considered to be the simplest known form of nonlinearity. Most optical fibers that are commercially available nowadays obey this Kerr law of nonlinearity. Therefore,in this first section the attention will be on optical metamaterials with cubic nonlinearity. In this case

$$F(u) = bu$$

for some non-zero constant b. Therefore, the governing equation given by (1) with Kerr law nonlinearity reduces to [2,10]

$$iq_t + aq_{xx} + b|q|^2q = i\alpha q_x + i\lambda\left(|q|^2q\right)_x + i\nu\left(|q|^2\right)_x q + \theta_1\left(|q|^2q\right)_{xx} + \theta_2|q|^2q_{xx} + \theta_3q^2q^*_{xx} \tag{5}$$

In order to locate the conservation laws for Eq. (5), the complex valued function is split into real and imaginary parts as:

$$q(x,t) = u(x,t) + iv(x,t).$$

This leads to the decomposition of Eq. (5) as

$$\begin{aligned}
&-\theta_2v^2u_{xx} - \theta_3u^2u_{xx} - 6\theta_1uu_x^2 - 2\theta_1uv_x^2 - \theta_2u^2u_{xx} - 3\theta_1u^2u_{xx}\\
&-\theta_1v^2u_{xx} + buv^2 + 2\nu v^2v_x + \theta_3v^2u_{xx} + \lambda u^2v_x + 3\lambda v^2v_x\\
&-4\theta_1vu_xv_x - 2\theta_1uvv_{xx} + 2\nu uvu_x - 2\theta_3uvv_{xx} + 2\lambda uvu_x\\
&-v_t + bu^3 + \alpha v_x + au_{xx} = 0, -\alpha u_x + bv^3 + av_{xx} - \theta_2u^2v_{xx}\\
&-\theta_2v^2v_{xx} - 2\theta_1vu_x^2 - 6\theta_1vv_x^2 - \theta_1u^2v_{xx} - 3\theta_1v^2v_{xx} - \lambda v^2u_x\\
&-2\nu u^2u_x + \theta_3u^2v_{xx} - 3\lambda u^2u_x - \theta_3v^2v_{xx} + bu^2v - 2\theta_3uvu_{xx}\\
&-2\theta_1uvu_{xx} - 4\theta_1uu_xv_x - 2\lambda uvv_x - 2\nu uvv_x + u_t = 0.
\end{aligned} \tag{6}$$

The lengthy calculations for the system (6) lead to the following multipliers

$$\begin{aligned}
Q^1 &= -c_1v\left((\theta_1+\theta_2-\theta_3)u^2 + (\theta_1+\theta_2-\theta_3)v^2 - a\right)^{\frac{\theta_3-\theta_2+\theta_1}{\theta_1+\theta_2-\theta_3}}\\
Q^2 &= c_1u\left((\theta_1+\theta_2-\theta_3)u^2 + (\theta_1+\theta_2-\theta_3)v^2 - a\right)^{\frac{\theta_3-\theta_2+\theta_1}{\theta_1+\theta_2-\theta_3}}
\end{aligned} \tag{7}$$

For the multipliers (7), we can not obtain conservation laws of the system (6) because of only implemented for integer exponents.

<u>Case-1</u>:

If we set $\theta_1 = \theta_2 = \theta_3 = 1$ in Eq. (5), for example, we have the following multipliers and corresponding conserved vectors

$$\begin{aligned}
Q^1 &= v(-u^2 - v^2 - a),\\
Q^2 &= -u(-u^2 - v^2 + a),
\end{aligned} \tag{8}$$

$$\begin{aligned}
T^t &= -\frac{(u^2+v^2)(-u^2-v^2+2a)}{4},\\
T^x &= \frac{(u^2+v^2)(-u^2-v^2+2a)\alpha}{4}\\
&\quad+\frac{(u^2+v^2)^2(-2u^2-2v^2+3a)\lambda}{4}\\
&\quad+\frac{(u^2+v^2)^2(-2u^2-2v^2+3a)\nu}{6} + v(-u^2-v^2+a)^2u_x\\
&\quad-u(-u^2-v^2+a)^2v_x
\end{aligned} \tag{9}$$

Thus, the conserved density for (5) is

$$\Phi^t = -\frac{|q|^2(-|q|^2+2a)}{4} \tag{10}$$

<u>Case-2</u>:

If we set $\theta_1 = 3, \theta_2 = 1, \theta_3 = 2$ in Eq. (5), for example, we have the following multipliers and corresponding conserved vectors

$$\begin{aligned}
Q^1 &= -v(-2u^2 - 2v^2 + a)^2,\\
Q^2 &= u(-2u^2 - 2v^2 + a)^2,
\end{aligned} \tag{11}$$

$$\begin{aligned}
T^t &= \frac{(u^2+v^2)(4u^4+8u^2v^2+4v^4-6au^2-6av^2+3a^2)}{6},\\
T^x &= -\frac{(u^2+v^2)(4u^4+8u^2v^2+4v^4-6au^2-6av^2+3a^2)\alpha}{6}\\
&\quad-\frac{(u^2+v^2)^2(6u^4+12u^2v^2+6v^4-8au^2-8av^2+3a^2)\lambda}{4}\\
&\quad-\frac{(u^2+v^2)^2(6u^4+12u^2v^2+6v^4-8au^2-8av^2+3a^2)\nu}{6}\\
&\quad-v(-2u^2-2v^2+a)^3u_x + u(-2u^2-2v^2+a)^3v_x
\end{aligned} \tag{12}$$

and

$$\Phi^t = \frac{|q|^2(4|q|^4 - 6a|q|^2 + 3a^2)}{6}. \tag{13}$$

Case-3:

If we set $\theta_1 = 2, \theta_2 = 1, \theta_3 = 2$ in Eq. (5), for example, we have the following multipliers and corresponding conserved vectors

$$Q^1 = v\left(-u^2 - v^2 + a\right)^3, \quad Q^2 = -u\left(-u^2 - v^2 + a\right)^3, \tag{14}$$

$$\begin{aligned} T^t &= -\frac{(u^2+v^2)(-u^2-v^2+2a)(u^4+2u^2v^2+v^4-2au^2-2av^2+2a^2)}{8}, \\ T^x &= \frac{(u^2+v^2)(-u^2-v^2+2a)(u^4+2u^2v^2+v^4-2au^2-2av^2+2a^2)\alpha}{8} \\ &+ \frac{3(u^2+v^2)^2(-4u^6-12u^4v^2-12u^2v^4-4v^6+15au^4+30au^2v^2+15av^4-20a^2u^2-20a^2v^2+10a^3)\lambda}{40} \\ &+ \frac{(u^2+v^2)^2(-4u^6-12u^4v^2-12u^2v^4-4v^6+15au^4+30au^2v^2+15av^4-20a^2u^2-20a^2v^2+10a^3)\nu}{20} + v\left(-u^2-v^2+a\right)^4 u_x - u\left(-u^2-v^2+a\right)^4 v_x \end{aligned} \tag{15}$$

and

$$\Phi^t = -\frac{|q|^2(-|q|^2+2a)(|q|^4-2a|q|^2+2a^2)}{8}. \tag{16}$$

Case-4:

If we set $\theta_1 = 5, \theta_2 = 1, \theta_3 = 4$ in Eq. (5), for example, we have the following multipliers and corresponding conserved vectors

$$Q^1 = -v\left(-2u^2 - 2v^2 + a\right)^4, \quad Q^2 = u\left(-2u^2 - 2v^2 + a\right)^4, \tag{17}$$

$$\begin{aligned} T^t &= \frac{(u^2+v^2)\left(16(u^2+v^2)^4 - 40a(u^2+v^2)^3 + 40a^2(u^2+v^2)^2 - 20a^3(u^2+v^2) + 5a^4\right)}{10} \\ T^x &= -\frac{(u^2+v^2)\left(16(u^2+v^2)^4 - 40a(u^2+v^2)^3 + 40a^2(u^2+v^2)^2 - 20a^3(u^2+v^2) + 5a^4\right)\alpha}{10} \\ &- \frac{(u^2+v^2)^2\left(80(u^2+v^2)^4 - 192a(u^2+v^2)^3 + 180a^2(u^2+v^2)^2 - 80a^3(u^2+v^2) + 15a^4\right)\lambda}{20} \\ &- \frac{(u^2+v^2)^2\left(80(u^2+v^2)^4 - 192a(u^2+v^2)^3 + 180a^2(u^2+v^2)^2 - 80a^3(u^2+v^2) + 15a^4\right)\nu}{30} - v\left(-2u^2-2v^2+a\right)^5 u_x + u\left(-2u^2-2v^2+a\right)^5 v_x \end{aligned} \tag{18}$$

and

$$\Phi^t = \frac{|q|^2\left(16|q|^8 - 40a|q|^6 + 40a^2|q|^4 - 20a^3|q|^2 + 5a^4\right)}{10}. \tag{19}$$

Thus, we have infinitely many conservation laws of the system (6).

The 1-soliton solution to the perturbed NLSE (5) is now given by [2,10]

$$q(x,t) = A\,\mathrm{sech}[B(x-ct)]e^{i(-\kappa x + \omega t + \theta_0)}, \tag{20}$$

where A is the amplitude of the soliton, B is the inverse width of the soliton. From the phase component, κ is the soliton frequency, ω is the wave number of the soliton and θ_0 is the phase constant. The speed of the soliton is given by

$$c = -\alpha - 2a\kappa. \tag{21}$$

For the four cases discussed above, the conserved quantities are now given by:

$$\begin{aligned} I_1 &= \int_{-\infty}^{\infty}\left(2a|q|^2 - |q|^4\right)dx \\ &= A^2\int_{-\infty}^{\infty}\left(2a\,\mathrm{sech}^2\tau - A^2\,\mathrm{sech}^4\tau\right)dx = \frac{4A^2}{3B}\left(3a - A^2\right), \end{aligned} \tag{22}$$

where the notation

$$\tau = B(x - ct) \tag{23}$$

is adopted here.

$$\begin{aligned} I_2 &= \int_{-\infty}^{\infty}\left(3a^2|q|^2 - 6a|q|^4 + 4|q|^6\right)dx \\ &= A^2\int_{-\infty}^{\infty}\left(3a^2\,\mathrm{sech}^2\tau - 6aA^2\,\mathrm{sech}^4\tau + 4A^4\,\mathrm{sech}^6\tau\right)dx \\ &= \frac{2A^2}{15B}\left(45a^2 - 60aA^2 + 32A^4\right) \end{aligned} \tag{24}$$

$$\begin{aligned} I_3 &= \int_{-\infty}^{\infty}|q|^2\left(2a - |q|^2\right)\left(2a^2 - 2a|q|^2 + |q|^4\right)dx \\ &= \int_{-\infty}^{\infty}\left(4a^3|q|^2 - 6a^2|q|^4 + 4a|q|^6 - |q|^8\right)dx \\ &= A^2\int_{-\infty}^{\infty}\left(4a^3\,\mathrm{sech}^2\tau - 6a^2A^2\,\mathrm{sech}^4\tau + 4aA^4\,\mathrm{sech}^6\tau - A^6\,\mathrm{sech}^8\tau\right)dx \\ &= \frac{8A^2}{35B}\left(35a^3 - 35a^2A^2 + 280aA^4 - 4A^6\right) \end{aligned} \tag{25}$$

$$I_4 = \int_{-\infty}^{\infty} \left(5a^4|q|^2 - 20a^3|q|^4 + 40a^2|q|^6 - 40a|q|^8 + 16|q|^{10}\right)dx$$
$$= A^2 \int_{-\infty}^{\infty} \left(5a^4 \operatorname{sech}^2\tau - 20a^3A^2 \operatorname{sech}^4\tau + 40a^2A^4 \operatorname{sech}^6\tau\right.$$
$$\left. - 40aA^6 \operatorname{sech}^8\tau + 16A^8 \operatorname{sech}^{10}\tau\right)dx = \frac{2A^2}{315B}\left(1575a^4 - 12600a^3A^2\right.$$
$$\left. + 6720a^2A^4 - 5760aA^6 + 2048A^8\right) \tag{26}$$

Parabolic law

This law is alternatively known as the cubic–quintic nonlinearity and is studied in nonlinear interaction between Langmuir waves and electrons. It describes the nonlinear interaction between the high frequency Langmuir waves and the ion-acoustic waves by pondermotive forces. It takes the form

$$F(u) = b_1 u + b_2 u^2$$

for non-zero constants b_1 and b_2. For parabolic law medium, the NLSE given by (1) changes to [10]

$$iq_t + aq_{xx} + \left(b_1|q|^2 + b_2|q|^4\right)q = i\alpha q_x + i\lambda\left(|q|^2 q\right)_x$$
$$+ i\nu\left(|q|^2\right)_x q + \theta_1\left(|q|^2 q\right)_{xx} + \theta_2|q|^2 q_{xx} + \theta_3 q^2 q^*_{xx} \tag{27}$$

Eq. (27) splits as

$$\theta_3 u_{xx}v^2 - 3\theta_1 u_{xx}u^2 - \theta_1 u_{xx}v^2 - \theta_2 u_{xx}u^2 - \theta_2 u_{xx}v^2 - \theta_3 u_{xx}u^2$$
$$+ au_{xx} + b_2u^5 + b_1u^3 + \alpha v_x - v_t + 2\lambda u v u_x + 2\nu u v u_x - 4\theta_1 v u_x v_x$$
$$- 2\theta_3 v_{xx}uv - 2\theta_1 v_{xx}uv + \lambda u^2 v_x + 3\lambda v^2 v_x + 2\nu v^2 v_x + 2b_2u^3v^2$$
$$+ b_2uv^4 + b_1uv^2 - 6\theta_1 uu_x^2 - 2\theta_1 uv_x^2 = 0,$$
$$- \alpha u_x + b_1v^3 + av_{xx} + b_2v^5 - \theta_2u^2v_{xx} - \theta_2v^2v_{xx} - 6\theta_1 vv_x^2$$
$$- 3\theta_1 v^2v_{xx} - 2\theta_1 vu_x^2 - \lambda v^2u_x - 2\nu u^2u_x - \theta_1u^2v_{xx} + 2b_2u^2v^3$$
$$- 3\lambda u^2u_x - \theta_3v^2v_{xx} + \theta_3u^2v_{xx} + b_1u^2v + b_2u^4v - 2\theta_3 uvu_{xx}$$
$$- 4\theta_1 uu_xv_x - 2\nu uvv_x - 2\theta_1 uvu_{xx} - 2\lambda uvv_x + u_t = 0 \tag{28}$$

For the system (28), we obtain Eqs. (7)–(19).

For parabolic law of nonlinearity, the bright 1-soliton solution in optical metamaterials is given by [10]:

$$q(x,t) = \frac{A}{\sqrt{D + \cosh[B(x - ct)]}} e^{i(-\kappa x + \omega t + \theta_0)}, \tag{29}$$

where D is a newly introduced parameter whose restriction will be given later. The remaining parameters have the same interpretations as Kerr law soliton. The conservation laws for parabolic law nonlinearity are now given by

$$I_1 = \int_{-\infty}^{\infty} \left(2a|q|^2 - |q|^4\right)dx$$
$$= A^2 \int_{-\infty}^{\infty} \left\{\frac{2a}{D + \cosh\tau} - \frac{A^2}{(D + \cosh\tau)^2}\right\}dx$$
$$= \frac{2A^2}{3B}\left\{6aF\left(1,1;\frac{3}{2};\frac{1-D}{2}\right) - A^2F\left(2,2;\frac{5}{2};\frac{1-D}{2}\right)\right\} \tag{30}$$

$$I_2 = \int_{-\infty}^{\infty} \left(3a^2|q|^2 - 6a|q|^4 + 4|q|^6\right)dx$$
$$= A^2 \int_{-\infty}^{\infty} \left\{\frac{3a^2}{D + \cosh\tau} - \frac{6aA^2}{(D + \cosh\tau)^2} + \frac{4A^4}{(D + \cosh\tau)^3}\right\}dx$$
$$= \frac{2A^2}{15B}\left\{45a^2F\left(1,1;\frac{3}{2};\frac{1-D}{2}\right) - 30aA^2F\left(2,2;\frac{5}{2};\frac{1-D}{2}\right)\right.$$
$$\left. + 8A^4F\left(3,3;\frac{7}{2};\frac{1-D}{2}\right)\right\} \tag{31}$$

$$I_3 = \int_{-\infty}^{\infty} |q|^2\left(2a - |q|^2\right)\left(2a^2 - 2a|q|^2 + |q|^4\right)dx$$
$$= \int_{-\infty}^{\infty} \left(4a^3|q|^2 - 6a^2|q|^4 + 4a|q|^6 - |q|^8\right)dx$$
$$= A^2 \int_{-\infty}^{\infty} \left\{\frac{4a^3}{D + \cosh\tau} - \frac{6a^2A^2}{(D + \cosh\tau)^2} + \frac{4aA^4}{(D + \cosh\tau)^3}\right.$$
$$\left. - \frac{A^6}{(D + \cosh\tau)^4}\right\}dx = \frac{4A^2}{105B}\left\{210a^3F\left(1,1;\frac{3}{2};\frac{1-D}{2}\right)\right.$$
$$- 105a^2A^2F\left(2,2;\frac{5}{2};\frac{1-D}{2}\right) + 28aA^4F\left(3,3;\frac{7}{2};\frac{1-D}{2}\right)$$
$$\left. - 3A^6F\left(4,4;\frac{9}{2};\frac{1-D}{2}\right)\right\} \tag{32}$$

$$I_4 = \int_{-\infty}^{\infty} \left(5a^4|q|^2 - 20a^3|q|^4 + 40a^2|q|^6 - 40a|q|^8 + 16|q|^{10}\right)dx$$
$$= A^2 \int_{-\infty}^{\infty} \left\{\frac{5a^4}{D + \cosh\tau} - \frac{20a^3A^2}{(D + \cosh\tau)^2} + \frac{40a^2A^4}{(D + \cosh\tau)^3}\right.$$
$$\left. - \frac{40aA^6}{(D + \cosh\tau)^4} + \frac{16A^8}{(D + \cosh\tau)^5}\right\}dx$$
$$= \frac{2A^2}{315B}\left\{1575a^4F\left(1,1;\frac{3}{2};\frac{1-D}{2}\right) - 2100a^3A^2F\left(2,2;\frac{5}{2};\frac{1-D}{2}\right)\right.$$
$$+ 1680a^2A^4F\left(3,3;\frac{7}{2};\frac{1-D}{2}\right) - 720aA^6F\left(4,4;\frac{9}{2};\frac{1-D}{2}\right)$$
$$\left. + 128A^8F\left(5,5;\frac{11}{2};\frac{1-D}{2}\right)\right\} \tag{33}$$

Here, in (30)–(33) Gauss' hypergeometric function is defined as:

$$F(\alpha,\beta;\gamma;z) = \sum_{n=0}^{\infty} \frac{(\alpha)_n(\beta)_n}{(\gamma)_n}\frac{z^n}{n!} \tag{34}$$

and the Pochhammer symbol is

$$(p)_n = \begin{cases} 1 & n = 0, \\ p(p+1)\cdots(p+n-1) & n > 0. \end{cases} \tag{35}$$

The convergence criteria for hypergeometric function is

$$|z| < 1. \tag{36}$$

which, for (30)–(33), implies

$$-1 < D < 3. \tag{37}$$

Furthermore, Rabbe's criteria of convergence implies

$$\gamma < \alpha + \beta, \tag{38}$$

which is valid for all of the hypergeometric functions listed above.

Conclusions

This paper retrieved conservation laws for solitons in optical metamaterials by the aid of Lie symmetry analysis. There are two forms of nonlinear media that are studied in this paper, which are Kerr law and parabolic law. Three of the important laws are not included in this paper since it is not possible to retrieve conserved densities for power law, dual-power law and logarithmic law by Lie symmetry. However, this paper is encouraging to seek for such densities for various other laws in the context of optical metamaterials. The results with those additional laws will be reported in future.

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