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Research Article

Coefficient Estimates for Two New Subclasses of Biunivalent Functions with respect to Symmetric Points

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We introduce two subclasses of biunivalent functions and find estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in these new subclasses. Also, consequences of the results are pointed out.

1. Introduction and Definitions

Let A denote the class of analytic functions in the unit disk

$$U = \{z \in \mathbb{C} : |z| < 1\} \tag{1}$$

that have the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{2}$$

Further, by S we will denote the class of all functions in A which are univalent in U.

The Koebe one-quarter theorem [1] states that the image of U under every function f from S contains a disk of radius $(1/4)$. Thus every such univalent function has an inverse f^{-1} which satisfies

$$\begin{gathered} f^{-1}(f(z)) = z, \quad (z \in U), \\ f\left(f^{-1}(w)\right) = w, \quad \left(|w| < r_0(f),\ r_0(f) \geq \frac{1}{4}\right), \end{gathered} \tag{3}$$

where

$$\begin{aligned} f^{-1}(w) = {} & w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 \\ & - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots . \end{aligned} \tag{4}$$

A function $f(z) \in A$ is said to be biunivalent in U if both $f(z)$ and $f^{-1}(z)$ are univalent in U. Let Σ denote the class of biunivalent functions defined in the unit disk U.

If the functions f and g are analytic in U, then f is said to be subordinate to g, written as

$$f(z) \prec g(z), \quad (z \in U) \tag{5}$$

if there exists a Schwarz function $w(z)$, analytic in U, with

$$w(0) = 0, \qquad |w(z)| < 1 \quad (z \in U) \tag{6}$$

such that

$$f(z) = g(w(z)) \quad (z \in U). \tag{7}$$

Lewin [2] studied the class of biunivalent functions, obtaining the bound 1.51 for modulus of the second coefficient $|a_2|$. Subsequently, Netanyahu [3] showed that $\max |a_2| = 4/3$ if $f(z) \in \Sigma$. Brannan and Clunie [4] conjectured that $|a_2| \leq \sqrt{2}$ for $f \in \Sigma$. Brannan and Taha [5] introduced certain subclasses of the biunivalent function class Σ similar to the familiar subclasses of univalent functions consisting of strongly starlike, starlike, and convex functions. They introduced bistarlike functions and obtained estimates on the initial coefficients. Bounds for the initial coefficients of several classes of functions were also investigated in [6–15].

Not much is known about the bounds on the general coefficient $|a_n|$ for $n \geq 4$. In the literature, there are only a few works determining the general coefficient bounds $|a_n|$ for the analytic biunivalent functions ([16–20]). The coefficient estimate problem for each of $|a_n|$ $(n \in \mathbb{N} \setminus \{1, 2\}; \mathbb{N} = \{1, 2, 3, \ldots\})$ is still an open problem.

By $S^*(\phi)$ and $C(\phi)$ we denote the following classes of functions:

$$S^*(\phi) = \left\{ f : f \in A, \frac{zf'(z)}{f(z)} \prec \phi(z); z \in U \right\},$$
$$C(\phi) = \left\{ f : f \in A, 1 + \frac{zf''(z)}{f'(z)} \prec \phi(z); z \in U \right\}. \tag{8}$$

The classes $S^*(\phi)$ and $C(\phi)$ are the extensions of classical sets of starlike and convex functions and in such form were defined and studied by Ma and Minda [21].

In [22], Sakaguchi introduced the class S_S^* of starlike functions with respect to symmetric points in U, consisting of functions $f \in A$ that satisfy the condition $\operatorname{Re}(zf'(z)/(f(z) - f(-z))) > 0$, $z \in U$. Similarly, in [23], Wang et al. introduced the class C_S of convex functions with respect to symmetric points in U, consisting of functions $f \in A$ that satisfy the condition $\operatorname{Re}((zf'(z))'/(f'(z) + f'(-z))) > 0$, $z \in U$. In the style of Ma and Minda, Ravichandran (see [24]) defined the classes $S_S^*(\phi)$ and $C_S(\phi)$.

A function $f \in A$ is in the class $S_S^*(\phi)$ if

$$\frac{2zf'(z)}{f(z) - f(-z)} \prec \phi(z), \quad z \in U, \tag{9}$$

and in the class $C_S(\phi)$ if

$$\frac{2\left(zf'(z)\right)'}{f'(z) + f'(-z)} \prec \phi(z), \quad z \in U. \tag{10}$$

In this paper, we introduce two new subclasses of biunivalent functions. Further, we find estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in these subclasses.

2. Coefficient Estimates for the Function Class $S^*_{S,\Sigma}(\alpha,h)$

Definition 1. Let the functions $h, p : U \to \mathbb{C}$ be so constrained that

$$\min\left\{\operatorname{Re}(h(z)), \operatorname{Re}(p(z))\right\} > 0,$$
$$h(0) = p(0) = 1. \tag{11}$$

Definition 2. A function $f \in \Sigma$ is said to be in the class $S^*_{S,\Sigma}(\alpha, h)$ if the following conditions are satisfied:

$$\frac{2\left[(1-\alpha) zf'(z) + \alpha z\left(zf'(z)\right)'\right]}{(1-\alpha)\left(f(z) - f(-z)\right) + \alpha z\left(f'(z) + f'(-z)\right)} \in h(U),$$
$$\frac{2\left[(1-\alpha) wg'(w) + \alpha w\left(wg'(w)\right)'\right]}{(1-\alpha)\left(g(w) - g(-w)\right) + \alpha w\left(g'(w) + g'(-w)\right)} \in p(U), \tag{12}$$

where $g(w) = f^{-1}(w)$.

Definition 3. One notes that, for $\alpha = 0$, one gets the class $S_S^*(h)$ which is defined as follows:

$$\frac{2zf'(z)}{f(z) - f(-z)} \in h(U),$$
$$\frac{2wg'(w)}{g(w) - g(-w)} \in p(U). \tag{13}$$

Theorem 4. *Let f given by (2) be in the class $S^*_{S,\Sigma}(\alpha, h)$. Then*

$$|a_2| \le \min\left\{\sqrt{\frac{\left|h'(0)\right|^2 + \left|p'(0)\right|^2}{8(1+\alpha)^2}}, \sqrt{\frac{\left|h''(0)\right| + \left|p''(0)\right|}{8(1+2\alpha)}}\right\},$$
$$|a_3| \le \min\left\{\frac{\left|h'(0)\right|^2 + \left|p'(0)\right|^2}{8(1+\alpha)^2} + \frac{\left|h''(0)\right| + \left|p''(0)\right|}{8(1+2\alpha)}, \frac{\left|h''(0)\right|}{4(1+2\alpha)}\right\}. \tag{14}$$

Proof. Let $f \in S^*_{S,\Sigma}(\alpha, h)$ and g be the analytic extension of f^{-1} to U. It follows from (12) that

$$\frac{2\left(zf'(z) + \alpha z^2 f''(z)\right)}{(1-\alpha)\left(f(z) - f(-z)\right) + \alpha z\left(f'(z) + f'(-z)\right)} = h(z), \quad (z \in U),$$
$$\frac{2\left(wg'(w) + \alpha w^2 g''(w)\right)}{(1-\alpha)\left(g(w) - g(-w)\right) + \alpha w\left(g'(w) + g'(-w)\right)} = p(w), \quad (w \in U), \tag{15}$$

where $h(z)$ and $p(w)$ satisfy the conditions of Definition 1. Furthermore, the functions $h(z)$ and $p(w)$ have the following Taylor-Maclaurin series expansions:

$$h(z) = 1 + h_1 z + h_2 z^2 + \cdots, \tag{16}$$

$$p(w) = 1 + p_1 w + p_2 w^2 + \cdots, \tag{17}$$

respectively. From (15), we deduce

$$2(1+\alpha) a_2 = h_1, \tag{18}$$

$$2(1+2\alpha) a_3 = h_2, \tag{19}$$

$$-2(1+\alpha) a_2 = p_1, \tag{20}$$

$$2(1+2\alpha)\left(2a_2^2 - a_3\right) = p_2. \tag{21}$$

From (18) and (20) we obtain

$$h_1 = -p_1, \tag{22}$$

$$8(1+\alpha)^2 a_2^2 = h_1^2 + p_1^2. \tag{23}$$

By adding (19) to (21), we get

$$4(1+2\alpha)a_2^2 = h_2 + p_2. \tag{24}$$

Therefore, we find from (23) and (24) that

$$\begin{aligned} |a_2|^2 &\le \frac{|h'(0)|^2 + |p'(0)|^2}{8(1+\alpha)^2}, \\ |a_2|^2 &\le \frac{|h''(0)| + |p''(0)|}{8(1+2\alpha)}. \end{aligned} \tag{25}$$

Subtracting (21) from (19) we have

$$4(1+2\alpha)a_3 - 4(1+2\alpha)a_2^2 = h_2 - p_2. \tag{26}$$

Then, upon substituting the value of a_2^2 from (23) and (24) into (26), it follows that

$$\begin{aligned} a_3 &= \frac{h_1^2 + p_1^2}{8(1+\alpha)^2} + \frac{h_2 - p_2}{4(1+2\alpha)}, \\ a_3 &= \frac{h_2 + p_2}{4(1+2\alpha)} + \frac{h_2 - p_2}{4(1+2\alpha)}. \end{aligned} \tag{27}$$

We thus find that

$$\begin{aligned} |a_3| &\le \frac{|h'(0)|^2 + |p'(0)|^2}{8(1+\alpha)^2} + \frac{|h''(0)| + |p''(0)|}{8(1+2\alpha)}, \\ |a_3| &\le \frac{|h''(0)|}{4(1+2\alpha)}. \end{aligned} \tag{28}$$

This completes the proof of Theorem 4. □

Taking $\alpha = 0$ we get the following.

Corollary 5. *If $f \in S_S^*(h)$ then*

$$\begin{aligned} |a_2| &\le \min\left\{\sqrt{\frac{|h'(0)|^2 + |p'(0)|^2}{8}}, \sqrt{\frac{|h''(0)| + |p''(0)|}{8}}\right\}, \\ |a_3| &\le \min\left\{\frac{|h'(0)|^2 + |p'(0)|^2}{8} + \frac{|h''(0)| + |p''(0)|}{8}, \frac{|h''(0)|}{4}\right\}. \end{aligned} \tag{29}$$

Corollary 6. *If we let*

$$\phi(z) = \left(\frac{1+z}{1-z}\right)^{\beta} = 1 + 2\beta z + 2\beta^2 z^2 + \cdots \quad (0 < \beta \le 1), \tag{30}$$

then inequalities (14) become

$$\begin{aligned} |a_2| &\le \min\left\{\frac{\beta}{1+\alpha}, \frac{\beta}{\sqrt{1+2\alpha}}\right\}, \\ |a_3| &\le \min\left\{\frac{\beta^2}{(1+\alpha)^2} + \frac{\beta^2}{1+2\alpha}, \frac{\beta^2}{1+2\alpha}\right\}. \end{aligned} \tag{31}$$

Corollary 7. *If we let*

$$\begin{aligned} \phi(z) &= \frac{1+(1-2\beta)z}{1-z} = 1 + 2(1-\beta)z \\ &\quad + 2(1-\beta)z^2 + \cdots \quad (0 \le \beta < 1), \end{aligned} \tag{32}$$

then inequalities (14) become

$$\begin{aligned} |a_2| &\le \min\left\{\frac{1-\beta}{1+\alpha}, \sqrt{\frac{1-\beta}{1+2\alpha}}\right\}, \\ |a_3| &\le \min\left\{\frac{(1-\beta)^2}{(1+\alpha)^2} + \frac{1-\beta}{1+2\alpha}, \frac{1-\beta}{1+2\alpha}\right\}. \end{aligned} \tag{33}$$

Remark 8. Corollaries 6 and 7 provide an improvement of the estimate $|a_3|$ obtained by Altınkaya and Yalçın [25].

Remark 9. The estimates on the coefficients $|a_2|$ and $|a_3|$ of Corollaries 6 and 7 are improvement of the estimates in [7].

3. Coefficient Estimates for the Function Class $£_{S,\Sigma}(\alpha,h)$

Definition 10. A function $f \in \Sigma$ is said to be $£_{S,\Sigma}(\alpha,h)$ if the following conditions are satisfied:

$$\begin{aligned} &\left(\frac{2zf'(z)}{f(z)-f(-z)}\right)^{\alpha}\left(\frac{2(zf'(z))'}{f'(z)+f'(-z)}\right)^{1-\alpha} \in h(z), \\ &\left(\frac{2wg'(w)}{g(w)-g(-w)}\right)^{\alpha}\left(\frac{2(wg'(w))'}{g'(w)+g'(-w)}\right)^{1-\alpha} \in p(w), \end{aligned} \tag{34}$$

where $g(w) = f^{-1}(w)$.

We note that, for $\alpha = 1$, the class $£_{S,\Sigma}(\alpha,h)$ reduces to the class $S_S^*(h)$.

Definition 11. One notes that, for $\alpha = 0$, one gets the class $C_S(h)$ which is defined as follows:

$$\begin{aligned} &\frac{2(zf'(z))'}{f'(z)+f'(-z)} \in h(U), \\ &\frac{2(wg'(w))'}{g'(w)+g'(-w)} \in p(U). \end{aligned} \tag{35}$$

Theorem 12. *Let f given by (2) be in the class $\pounds_{S,\Sigma}(\alpha, h)$. Then*

$$|a_2| \le \min\left\{\sqrt{\frac{|h'(0)|^2+|p'(0)|^2}{8(2-\alpha)^2}}, \sqrt{\frac{|h''(0)|+|p''(0)|}{8(3-3\alpha+\alpha^2)}}\right\}, \tag{36}$$

$$|a_3| \le \min\left\{\frac{|h'(0)|^2+|p'(0)|^2}{8(2-\alpha)^2}+\frac{|h''(0)|+|p''(0)|}{8(3-2\alpha)}, \frac{(6-5\alpha+\alpha^2)|h''(0)|+(\alpha-\alpha^2)|p''(0)|}{8(3-2\alpha)(3-3\alpha+\alpha^2)}\right\}. \tag{37}$$

Proof. Let $f \in \pounds_{S,\Sigma}(\alpha, h)$ and g be the analytic extension of f^{-1} to U. We have

$$\begin{aligned}
&\left(\frac{2zf'(z)}{f(z)-f(-z)}\right)^{\alpha}\left(\frac{2(zf'(z))'}{f'(z)+f'(-z)}\right)^{1-\alpha} \\
&= 1+2(2-\alpha)a_2z \\
&\quad + \left[2(3-2\alpha)a_3-2\alpha(1-\alpha)a_2^2\right]z^2+\cdots, \\
&\left(\frac{2wg'(w)}{g(w)-g(-w)}\right)^{\alpha}\left(\frac{2(wg'(w))'}{g'(w)+g'(-w)}\right)^{1-\alpha} \\
&= 1-2(2-\alpha)a_2w \\
&\quad + \left[2(3-2\alpha)(2a_2^2-a_3)-2\alpha(1-\alpha)a_2^2\right]w^2+\cdots.
\end{aligned} \tag{38}$$

It follows from (34) that

$$\begin{aligned}
&\left(\frac{2zf'(z)}{f(z)-f(-z)}\right)^{\alpha}\left(\frac{2(zf'(z))'}{f'(z)+f'(-z)}\right)^{1-\alpha} = h(z), \\
&\qquad (z \in U), \\
&\left(\frac{2wg'(w)}{g(w)-g(-w)}\right)^{\alpha}\left(\frac{2(wg'(w))'}{g'(w)+g'(-w)}\right)^{1-\alpha} = p(w), \\
&\qquad (w \in U),
\end{aligned} \tag{39}$$

where $h(z)$ and $p(w)$ satisfy the conditions of Definition 1.

From (39), we deduce

$$2(2-\alpha)a_2 = h_1 \tag{40}$$

$$2(3-2\alpha)a_3-2\alpha(1-\alpha)a_2^2 = h_2, \tag{41}$$

$$-2(2-\alpha)a_2 = p_1 \tag{42}$$

$$2(3-2\alpha)(2a_2^2-a_3)-2\alpha(1-\alpha)a_2^2 = p_2. \tag{43}$$

From (40) and (42) we obtain

$$h_1 = -p_1, \tag{44}$$

$$8(2-\alpha)^2a_2^2 = h_1^2+p_1^2. \tag{45}$$

By adding (41) to (43), we get

$$4(3-3\alpha+\alpha^2)a_2^2 = h_2+p_2, \tag{46}$$

which gives us the desired estimate on $|a_2|$ as asserted in (36).

Subtracting (43) from (41) we have

$$4(3-2\alpha)a_3-4(3-2\alpha)a_2^2 = h_2-p_2. \tag{47}$$

Then, in view of (45) and (46), it follows that

$$\begin{aligned}
a_3 &= \frac{h_1^2+p_1^2}{8(2-\alpha)^2}+\frac{h_2-p_2}{4(3-2\alpha)}, \\
a_3 &= \frac{h_2+p_2}{4(3-3\alpha+\alpha^2)}+\frac{h_2-p_2}{4(3-2\alpha)}
\end{aligned} \tag{48}$$

as claimed. This completes the proof of Theorem 12. □

Taking $\alpha = 0$ we get the following.

Corollary 13. *If $f \in C_S(h)$ then*

$$|a_2| \le \min\left\{\sqrt{\frac{|h'(0)|^2+|p'(0)|^2}{32}}, \sqrt{\frac{|h''(0)|+|p''(0)|}{24}}\right\}, \tag{49}$$

$$|a_3| \le \min\left\{\frac{|h'(0)|^2+|p'(0)|^2}{32} + \frac{|h''(0)|+|p''(0)|}{24}, \frac{|h''(0)|}{12}\right\}. \tag{50}$$

Corollary 14. *If we let*

$$\phi(z) = \left(\frac{1+z}{1-z}\right)^{\beta} = 1+2\beta z+2\beta^2z^2+\cdots \quad (0 < \beta \le 1), \tag{51}$$

then inequalities (36) and (37) become

$$\begin{aligned}
|a_2| &\le \min\left\{\frac{\beta}{2-\alpha}, \frac{\beta}{\sqrt{3-3\alpha+\alpha^2}}\right\}, \\
|a_3| &\le \min\left\{\frac{\beta^2}{(2-\alpha)^2}+\frac{\beta^2}{3-2\alpha}, \frac{\beta^2}{3-3\alpha+\alpha^2}\right\}.
\end{aligned} \tag{52}$$

Corollary 15. *If we let*

$$\begin{aligned}
\phi(z) &= \frac{1+(1-2\beta)z}{1-z} = 1+2(1-\beta)z \\
&\quad + 2(1-\beta)z^2+\cdots \quad (0 \le \beta < 1),
\end{aligned} \tag{53}$$

then inequalities (36) and (37) become

$$\left|a_2\right| \leq \min\left\{\frac{1-\beta}{2-\alpha}, \sqrt{\frac{1-\beta}{3-3\alpha+\alpha^2}}\right\},$$
$$\left|a_3\right| \leq \min\left\{\frac{(1-\beta)^2}{(2-\alpha)^2}+\frac{1-\beta}{3-2\alpha}, \frac{1-\beta}{3-3\alpha+\alpha^2}\right\}. \tag{54}$$

Remark 16. Corollaries 14 and 15 provide an improvement of the estimate $|a_3|$ obtained by Altınkaya and Yalçın [25].

Remark 17. The estimates on the coefficients $|a_2|$ and $|a_3|$ of Corollaries 14 and 15 are improvement of the estimates obtained in [7].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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