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A Psychometric Approach to the VIKOR Method for Eliciting Subjective Public Assessments

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ABSTRACT Based on the concepts of sensory thresholds, the Weber-Fechner Law in psychology has usually been applied to sensory dimensions. However, some neurophysiological studies have shown that assessment of mental numbers or numerical information also follows this law. Therefore this paper proposes a modification of VIKOR that reflects the Weber-Fechner law to account for nonlinearity in evaluation scales as perceived by decision-makers. The model is applied to a case where public approval of two different types of public bus operation systems considering six criteria is pursued. The results are compared with those obtained from the multi-attribute value method (MAVT). The suggested method inflates the relative attractiveness of the alternatives that are closer to the best solutions (used by VIKOR). A numerical example is also provided to illustrate the applicability of the approach. This method can be a useful tool where public opinions based on subjective perceptions in a public participation process are of particular interest rather than assessing verifiable facts, i.e., observable, measurable data.

INDEX TERMS VIKOR, Weber-Fechner law, multi-attribute value theory, public subjective opinions, public decision-making.

I. INTRODUCTION

The public's subjective opinions on transportation projects and issues have become crucial, especially after the urge of public participation by the Intermodal Surface Transportation Efficiency Act (ISTEA) of 1991 [1]. It has been emphasized that public involvement in decision making is an integral component of transportation planning. The International Association for Public Participation (IAP2) [2] has further clarified the role of the public in the planning and decision-making process. The level of public participation is defined in a spectrum from the lowest to highest level as follows: inform (to provide the community with balanced and objective information to assist them in understanding the problem, possible alternatives/opportunities and solutions); consult (to obtain feedback from the community); involve (to work directly with the community throughout the process); collaborate (to partner with the community in all aspects of the decision from development to defining solutions); empower (to place final decision-making in the hands of the community) [2]. At the highest level of participation, it is expected that the public has a direct voice in the final decision-making process. This is particularly important to increase the legitimacy of projects and decisions. Moreover, it can elevate perspectives that are more closely aligned with those of the general public [3]. Therefore, in such cases disclosing actual subjective thoughts can provide greater insights into the real needs and concerns of the public. Recognizing those needs and concerns allows policymakers to either improve or revise the projects and policies. Moreover, policymakers can, at least, address the concerns to enhance the needed support for the implementation of the projects.

Although extensive research has been performed to develop effective and appropriate strategies for successful public participation practices (see, for instance, [4]–[7]), decision-making practices together with public involvement have remained limited and have not been well established yet. The State Street Corridor Strategic Plan Study of the City of Boise, WA, USA [8] can be given as an example of such a process of involving the public in decision making. Possible scenarios were assessed on a given 5-point scale to realize the level of support for the plan. Although such a process provided useful information to balance the needs of the stakeholders and influenced the results and the outcome of the State Street Corridor Study, the method used to elicit public opinions did not consider the subjective nature of public judgments. Thus, a greater insight into the public's opinions such as their subjective preferences about the given

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scenarios could not be obtained and the study did not reach its ultimate goal.

Subjective preferences can be determined by a measurement technique or a combination of different techniques. Naturally, decisions among the alternatives involve several conflicting decision criteria to be dealt with. Therefore, revealing subjective preferences is the topic of the multi-criteria decision analyses (MCDA) domain. There are many MCDA methods to reveal decision-makers' preferences using different rational approaches and analytical structures for evaluating the alternatives. Among the most famous ones are the multi-attribute value theory (MAVT) by Keeney and Raiffa [9] and Dees *et al.* [10], the analytic hierarchy process (AHP) by Saaty [11], technique for order preference by similarity to ideal solution (TOPSIS) by Hwang and Yun [12], the elimination and choice expressing reality (ELECTRE) by Roy [13], the preference ranking organization method for enrichment evaluation (PROMETHEE) by Brans *et al.* [14] and Multi-criteria Optimization and Compromise Solution (VIKOR) by Opricovic [15]. These methods and/or their variants have been widely implemented in a variety of domains to elucidate opinions in the form of either ranking or preferences. Although these methods aim to capture subjective judgments, the cognitive aspects of human behavioral psychology for structuring cognitive assessments and decision-making processes are not integrated [16].

This paper is intended to present a decision-support system that uses behavioral theories to account for human cognitive process to elicit subjective judgments of the public involved in the decision making process. In this respect, using VIKOR's sound theoretical basis, the Weber–Fechner psycho-physical law in behavioral psychology is adopted to reflect perceptual discrimination for assessing the alternatives concerning a given set of criteria. Moreover, by adopting this law, the suggested VIKOR can also be converted to a preference measurement model that can not only provide the rankings of alternatives but also capture the subjective preferences of decision makers.

Thus, the main contribution of this paper lies in incorporating the Weber-Fechner psycho-physical law into VIKOR to integrate human perceptual discrimination behavior in a decision-support system. The findings of the study show that this proposed method is robust and reliable and therefore can be used in cases where public opinions based on subjective perceptions are of particular interest.

The proposed method was applied to a case where the public's approval of two different types of public bus operation systems considering six criteria is sought. A numerical example drawn from the data was also provided to illustrate the applicability of the model. Moreover, a sensitivity analysis was also performed to evaluate the influence of the thresholds representing noticeable changes adopted from the Weber-Fechner law.

The organization of the paper is as follows: the theoretical background is given in Section 2, where the theories of VIKOR, Weber-Fechner law and MAVT method are explained in detail. Then, the methodology is given in Section 3, where the proposed VIKOR method is explained. An illustrative example together with the overall results obtained from the data set used for the study is provided in Section 4. In relation to the results, the discussions based on the crucial findings are also given in this section. Finally, conclusions are drawn and suggestions for further practice and research are given in Section 5.

II. THEORETICAL BACKGROUND

A. VIKOR METHOD

The VIKOR method (the Serbian name, VlseKriterijumska Optimizacija I Kompromisno Resenje) is a relatively new multi-criteria optimization method developed by Opricovic [15] for solving complex systems. It 'first focuses on ranking alternatives and then applies an optimization technique to define a set of alternatives that offers a compromise solution to a problem having conflicting criteria.

Despite being a relatively new method, VIKOR or its combinations with other techniques have found many applications in a variety of domains. Some recent ones include waste management by Liu *et al.* [17], strategic environmental assessments by Kim *et al.* [18], prioritizing pavement maintenance activities by Babashamsi *et al.* [19], performance evaluation of railway zones by Ranjan *et al.* [20], ranking causes of delay in a metro system by Hajiagha *et al.* [21], evaluating intelligent transportation systems by Dong *et al.* [22], assessing sustainable transport fleet by Bai *et al.* [23], and about solving multiple-attribute decision making (MADM) problems in the maritime transportation industry by Soner *et al.* [24]. A complete systematic state of the art literature review for VIKOR is provided by Mardani *et al.* [25].

The performances of alternatives are evaluated for each criterion by a performance function that takes into account ratings' closeness to ideal solutions [15], [26]–[28]. In VIKOR, a multi-criteria decision-making problem can be defined in matrix form as:

$$
\mathbf{D} = \begin{array}{c} \\ A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array}
\begin{array}{c}
\begin{array}{ccccc} C_1 & \cdots & C_j & \cdots & C_n \end{array} \\
\begin{bmatrix}
f_{11} & \cdots & f_{1j} & \cdots & f_{1n} \\
\vdots & \ddots & \vdots & \vdots & \vdots \\
f_{i1} & \cdots & f_{ij} & \cdots & f_{in} \\
\vdots & \cdots & \vdots & \ddots & \vdots \\
f_{m1} & \cdots & f_{mj} & \cdots & f_{mn}
\end{bmatrix}
\end{array} \tag{1}
$$

where $\mathbf{D}$ is the decision matrix, and A_i denotes the alternatives ($i = 1, 2, \ldots m$), whereas C_j shows the criteria ($j = 1, 2, \ldots n$). The values of the $\mathbf{D}$ matrix, f_{ij}, indicate the performance rating of alternative A_i with respect to criterion C_j. The VIKOR method is grounded on the following form of L_p-metric:

$$
L_{p,i} = \left\{ \sum\nolimits_{j=1}^{n} \left[w_j (f_j^* - f_{ij})/(f_j^* - f_j^-) \right]^p \right\}^{1/p}; \quad 1 \le p \le \infty; \quad i = 1, 2, \ldots, m \tag{2}
$$

$\boldsymbol{L_{1,i}}$ and $\boldsymbol{L_{\infty,i}}$ are used to formulate ranking measure, where $\boldsymbol{f_j^*}$ and $\boldsymbol{f_j^-}$ represent the best and worst solutions respectively and $\boldsymbol{w_j}$ represents the weights assigned for the criteria. The ordinary VIKOR has the following steps:

(a) Defining the best f_j^* and worst solutions f_j^- of all criterion; $i = 1, 2, \ldots, m$.

$$f_j^* = \left\{ \left(\max_i f_{ij} | j \in I' \right), \min_i f_{ij} | j \in I'' \right\} \tag{3}$$

$$f_j^- = \left\{ \left(\min_i f_{ij} | j \in I' \right), \max_i f_{ij} | j \in I'' \right\} \tag{4}$$

where I' is the set of benefit criteria (maxima for f^* and minima for f^-) and I'' is the set of cost criteria (minima for f^* and maxima for f^-).

(b) Computing S_i and R_i from the following equations:

$$S_i = \sum_{j=1}^{n} w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-) \tag{5}$$

$$R_i = \max_i \left[w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-) \right] \tag{6}$$

where w_j are the weights of criteria. It should be noted that S_i and R_i correspond to $L_{1,i}$ and $L_{\infty,i}$, respectively. As recognized, R_i is only a part of S_i; therefore, it is always less than or equal to S_i and may sometimes provide unexpected results with respect to S_i solutions (see the discussion related to the analysis given in Section 4).

(c) Computing the values of Q_i by the following equations (if a compromise solution is sought):

$$Q_i = \frac{\vartheta \,(S_i - S^*)}{S^- - S^*} + \frac{(1-\vartheta)\,(R_i - R^*)}{R^- - R^*} \tag{7}$$

where

$$S^* = \min_i S_i \quad S^- = \max_i S_i \tag{8}$$

$$R^* = \min_i R_i \quad R^- = \max_i R_i \tag{9}$$

where ϑ is defined as the weight of the strategy of "the majority of criteria", it is usually accepted as 0.5 indicating "by consensus" (when $\vartheta > 0.5$ "by majority rule" and $\vartheta < 0.5$ "with veto"). This last step of calculations is needed for obtaining a compromised solution based on the specified weight of the strategy. It represents a decision maker's negotiations on the preferences of criteria weights.

(d) Ranking the alternatives by sorting the values of S, R, and Q in increasing order.

It should be noted that the rankings obtained from S (5), R (6) and Q (7) can be incompatible depending on the specified weight of the strategy.

The ordinary VIKOR is an effective decision support methodology particularly if a decision-maker has difficulty expressing his/her preferences at the beginning of a decision-making process [26]–[28]. However, this is not the case considered in this paper for the reasons stated in the following paragraph and Section 3, where the suggested method is explained. Therefore, the compromising approach is not considered in this paper. Hence, the additional rules that are set for obtaining valid compromised solutions in the ordinary VIKOR method are neither included nor discussed in this paper.

Despite its popularity, the method has its drawbacks particularly for offering a compromise solution that can be obtained from (7). For instance, let us consider a straightforward case where two alternatives are evaluated concerning two benefit criteria with equal weights:

$$\boldsymbol{D} = \begin{array}{c c} & \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 20 & 70 \\ 70 & 20 \end{bmatrix} \end{array}$$

where $f_1^* = f_2^* = 70$ and $f_1^- = f_2^- = 20$, therefore $S^* = S^- = 1.0$ and $R^- = R^* = 1.0$. It is clear that (7) will return a mathematical error due to division by zero. It can be even worse if all f_{ij} are equal to each other, since in that case (5) and (6) will also end up with mathematical errors. Chang [29] offered a modified VIKOR method to avoid those numerical difficulties in solving problems with the traditional VIKOR method. One of the solutions that he has offered is to eliminate meaningless criteria. Although the solution sounds reasonable, defining meaningless criteria can be difficult by using (5) and (6), since multiplying criteria weights by ratings with different units can create difficulty in deciding which criteria are meaningless. Therefore, particularly the ranking of Q obtained from (7) is recognized to be irrelevant in assessing subjective opinions that are pursued in this paper.

B. WEBER-FECHNER PSYCHOMETRIC SCALE

The Weber-Fechner psycho-physical law relates actual stimulus intensity to perceptual intensity by a psychophysical scale. The law tries to explain subjective values or perceptions in mind as a function of physical sensations. The general relationship between perceived sensation and external stimuli was first explored by Weber [30]. A just noticeable difference (*jnd*) is defined as the smallest difference in the intensity of two stimuli that can be detectable by subjects:

$$\Delta f = \epsilon f \quad \text{or } \Delta f / f = \epsilon \tag{10}$$

where Δf represents the *jnd*, f represents the initial stimulus intensity, and ϵ is a ratio called the Weber fraction. This law implies that perceived intensity is proportional to physical stimuli. In other words, the ability of a person to distinguish the magnitude of two stimuli is managed not by the absolute difference between the stimuli but rather by the ratio of the difference between the two stimuli [31]–[33]. For a practical example, let us consider a bus service that usually arrives at a station with a one-minute delay. If the bus reaches the station with a 1.50-minute delay, the delay will easily be noticed by most people. However, if the bus usually arrives with a 5-minute delay, a 5.50-minute delay will not easily be noticed by most people regardless of the same absolute difference (0.50 minutes or 30 seconds). According to the Weber law, the 5.50 minutes have a minimal change relative to the intensity of the original 5 minutes. However, the relative change from 1 minute to 1.50 minutes is much greater; therefore, the change exceeds the just noticeable difference. What Weber suggests is that *jnd*s are directly proportional to the "intensity" of the stimulus, or the usual arrival time in this example.

Fechner [34] further extended Weber's law and suggested a theoretical explanation for Weber's observations that became a psycho-physical scale relating the intensity of a stimulus, f, to a subjective perception. Psychologically perceived intensity to the physical intensity of a stimulus is represented in mathematical form as:

$$K = \epsilon \cdot log\left(\frac{f}{f_0}\right) \tag{11}$$

where K represents one's perception or psychological sensation of a change for the initial stimulus f_0 and added stimulus $f(= f_0 + \Delta f)$. It implies that the relationship between perception and stimulus is logarithmic. Although, Fechner called this relation "Weber's law", it has been called the "Weber-Fechner law" by psychologists [35]. The Weber-Fechner law is also consistent with a continuum of sensation [34], [36].

The Weber-Fechner law has arguably been challenged by Stevens' power law [37] in psychology. Stevens' power law specifies the relationship between the intensity of sensation as $S = kf^a$ where S is the sensation magnitude, f is the magnitude of physical stimulus, k is a scaling factor and a is the exponent that is empirically estimated and may differ among senses [35]. However, many researchers suggest that the psycho-physical predictions of these models are necessarily equivalent [36], [38]. Moreover, Sun *et al.* [39] also revealed that the Weber-Fechner law could well approximate the sensations corresponding to many natural phenomena having statistical distributions that obey a power-law over a range of intensities.

According to the Weber-Fechner law, the size of a *jnd* is proportional to the actual stimulus intensity value and perception is logarithmic to stimulus. In other words, subjective equality increases as the logarithm of the stimulus. The relation between intensities over a specific range constitutes a sequence with geometric progression as:

$$\begin{aligned} f_1 &= f_0 + \epsilon f_0 = (1+\epsilon)^1 f_0; \quad f_2 = f_1 + \epsilon f_1 \\ &= (1+\epsilon)^2 f_0; \ldots; f_v \\ &= f_{v-1} + \epsilon f_{v-1} = (1+\epsilon)^v f_0 \end{aligned} \tag{12}$$

It implies that *jnd*s are perceptually equal (subjective equality), or in other words, all *jnds* produce the same amount of change in perception. Thus;

$$\begin{aligned} \Delta f_v &= (1+\epsilon)^1 \Delta f_{v-1} = (1+\epsilon)^2 \Delta f_{v-2} \\ &= \cdots = (1+\epsilon)^v \Delta f_0 \end{aligned} \tag{13}$$

where Δf_0 is the initial step or the first notice over a range of $f_{max} - f_{min}$:

$$\Delta f_0 = \frac{f_{max} - f_{min}}{(1+\epsilon)^v} \tag{14}$$

where $(1 + \epsilon)$ is the progression factor and v is an integer showing the number of thresholds in the stimulus or the number of *jnd*s (that is $v + 1$ intervals on the range of $f_{max} - f_{min}$) that causes a change in sensation.

The idea of *jnd*s is adopted in this paper to characterize the psychometric scale that one uses for measuring distances to the best or worst solution for comparison judgments. Therefore, the *jnd*s represent threshold values corresponding to a unit change of sensation in the context of subjective judgment. Accordingly, the Weber-Fechner law implies that as two alternatives, A and B, are compared over $f_{min} - f_{max}$ (or $f_j^- - f_j^*$ in VIKOR) a unit difference between two alternatives $(A - B)$ closer to f_{min} does not have the same effect on judgment as a unit difference closer to f_{max} (see Fig. 2 for a schematic representation of *jnd*s that is shown on the abscissa for the example given in Section 4).

Based on the Weber-Fechner law, the range of the best f_j^* and worst f_j^- solutions of VIKOR can be divided into subjectively equal parts that represent just noticeable differences in the range. Then, applying the suggested method as explained in Section 3, VIKOR can be converted into a value measurement model, e.g., the multi-attribute utility model (MAUT).

C. MAVT METHOD

A simple additive multi-attribute value method [9] (MAVT) assumes a linear value function for all criteria. The MAVT method is utilized in this paper for two reasons: first, it is simple to use and well-accepted as a valid method that the suggested method can be compared with (in terms of ranking and preferences), and second, the ordinary VIKOR is a MAVT method as explained in the following paragraphs.

The MAVT determines the scores (or preferences) of alternatives concerning the relative positions of ratings from the minimum score on the range of max-min scores. The MAVT uses the following value function for defining preferences:

$$p_i(f_{ij}) = \frac{f_{ij} - min_j(f_{ij})}{max_j(f_{ij}) - min_j(f_{ij})} \quad \text{for } i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n \tag{15}$$

As can be seen, analogous to the ordinary VIKOR method, the MAVT also implicitly considers a certain amount of distances between the ratings as equally important regardless of the relative locations to the best/worst. The total scores can be calculated from the arithmetic mean rule as defined:

$$p_i = \sum_{j=1}^{n} W_j p_{ij} \quad \text{for } i = 1, 2, \ldots, m \tag{16}$$

Preferences are multiplied by the criteria weights, W_j and summed up. It can be noted that the MAVT (15) and the VIKOR S formula Eq. (5) use the same concept but approach the solution differently. While the MAVT considers the distance to the minimum that is $(f_{ij} - f_j^-)/(f_j^* - f_j^-)$, VIKOR takes the other way around: $(f_j^* - f_{ij})/(f_j^* - f_j^-)$. Moreover, while the MAVT sorts the alternatives in decreasing order, VIKOR unsurprisingly sorts the alternatives in increasing order to provide the preferred rankings. Therefore, it will not be astonishing to find that both methods yield compatible results in ranking. The relation of the S of VIKOR to preference values in MAVT (15) will be:

$$p_{ij} = 1 - S_{ij} \quad \text{for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n \qquad (17)$$

These findings also support the idea that the suggested VIKOR method grounded on behavioral psychology can be extended to function as a value (preference) measurement model. Simply, p_{ij} are multiplied by the associated weights as defined in (16), and they are summed up and then normalized for obtaining preferences, P_i, among the alternatives as:

$$P_i = \frac{p_i}{\sum_i^m p_i} \quad \text{for } i = 1, 2, \ldots, m \qquad (18)$$

III. METHODOLOGY

The ordinary VIKOR method disregards perceptual discrimination behavior. In the cognitive process of decision making, perceptual discrimination refers to one's ability to discern the minimal perceived difference in stimulus intensity. However, the ordinary VIKOR rankings represent n-dimensional L_p-metric distances from the best f_j^* and worst solutions f_j^- (where n is the number of criteria considered). This implies that a unit difference in the range $f_j^* - f_j^-$ has equivalent impacts on preferences regardless of their relative locations to the best and the worst solutions. However, human perceptions are found to be approximately logarithmic [40]. In other words, mappings from observable stimulus to internal perception space (a psycho-physical scale that is used for judgments) are actually not linear as discussed in the Weber-Fechner law. Moreover, some neurophysiological studies have also shown that even mental numbers or numerical information seem to be logarithmic rather than linear [41]–[43]. Nieder and Miller [41], Dehaene [36], and Hannagan *et al.* [44] asserted that the law holds also for perception numerosity.

In this respect, the psycho-physical scale of the Weber-Fechner law can serve as a practical instrument to account for subjective/perceived ratings' closeness to the best/worst solutions to reflect the nonlinearity in perception. In order to value *jnd*s suitably, more weights can be assigned to *jnd*s closer to the best solution, f_j^*. In short, the suggested VIKOR method is grounded on this idea: a rating closer to f_j^* can be valued more by providing more weights to *jnd*s (defined as the intervals on the range over which performances are evaluated) closer to f_j^*, accordingly.

The ordinary VIKOR approach also fits well into this notion if the criteria weights in (5) and (6) are treated as weights of these *jnd*s (pieces of distances defined according to Weber-Fechner law).

The flowchart explaining the computational steps of the proposed method is presented in Fig. 1.

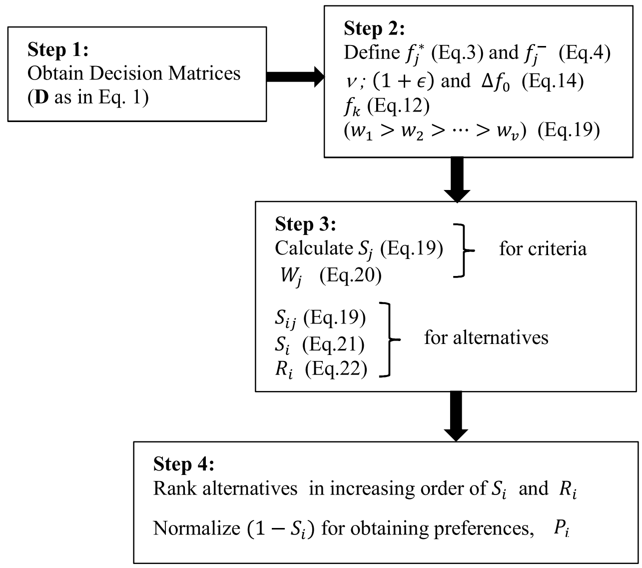

FIGURE 1. The computational steps of the proposed method.

Accordingly, in step 1 the decision matrices are obtained as in (1) of the ordinary VIKOR method. Decision matrices contain performance ratings of alternatives with respect to a set of criteria considered in the evaluation process. Likewise, matrices indicating the importance ratings of the criteria with respect to the objective are also obtained in this step. Then, in step 2, the best f_j^* and the worst f_j^- are acquired in the same way as defined in (3) and (4). As realized, the range of f_j^* and f_j^- for each criterion represents the universe of discourse over which alternatives are assessed. In this regard, f_j^* and f_j^- correspond to f_{max} and f_{min} in (14) of the Weber-Fechner law, respectively. Correspondingly, the ordinary VIKOR model is modified for accompanying the psychophysics law: the universe of discourse (defined as the range of $f_{max} - f_{min}$) can be divided into several subjectively equal subintervals that represent *jnd*s. In order to do this, the number of thresholds, ν, the progression factor, $(1+\epsilon)$, the noticeable first step, Δf_0, should be defined as in (14). Then, the threshold values f_k can be calculated from (12). Based on Weber's law, Lootsma [31], [45], [46] has shown and provided various examples where human beings follow a uniform pattern when they subdivide a particular range into subjectively equal intervals with a progression factor that is $(1+\epsilon) = 2$, which is defined in (14). Of course, it is not an arbitrary number since it implies that one's recognition starts with a small initial step from the one end of the range (the best solution) and follows the steps doubling each time until the other end (the worst solution) is reached. Naturally, assuming that this pattern is valid for all individuals would be naive but it can still be useful for representing asymmetric human cognitive appraisal for assessing the perception-based decision-making process. This assumption is similar to preferring a sigmoid or logistic function to map utilities to choice probabilities in the logit utility method where choices made by decision makers are observable (i.e., mode choices). The weights of *jnds* w_k $(w_1 > w_2 > \ldots > w_\nu)$ are also determined in this step.

In step 3, the solution for S (S_j for the criteria and S_i for the alternatives) can be obtained in two stages: first, Eq. (5) is modified to consider the weighted metric distances of rating f_{ij} to the best value f_j^* for each criterion j as:

$$S_{ij} = \begin{cases} w_1 \dfrac{(f_{ij} - f_0)}{(f_j^* - f_j^-)} \\ \text{if } f_1 \le f_{ij} \le f_0 \\ w_{k+1} \dfrac{(f_{ij} - f_k)}{(f_j^* - f_j^-)} + \displaystyle\sum_{r=0}^{k-1} w_{r+1} \dfrac{\Delta f_r}{(f_j^* - f_j^-)} \\ \text{if } f_{k+1} \le f_{ij} < f_k \text{ for } k = 1, 2, \ldots, v \end{cases} \tag{19}$$

where v is the number of the threshold, Δf_r are the noticeable differences (Δf_0 is the first noticeable difference), f_k are the threshold values and w_k are the weights ($w_1 > w_2 > \ldots > w_v$) assigned for subjectively equal intervals, *jnd*s, as described in step 2. For the sake of simplicity, the sum of the weighted distance to the best solution is shown as $\left(f_j^* - f_j^-\right)$ in (19) where it is actually $\left(f_j^* - f_j^-\right) = \sum_{k=0}^{v} w_k \Delta f_r$. Moreover, $f_{v+1} = f_{min}$ and $f_0 = f_{max}$ in (19). It should be noted that in order to calculate S_j for the criteria from (19), the S_{ij} should be replaced by S_j and f_{ij} should be replaced by f_j. Then, the normalized $(1 - S_j)$ values of criteria provide the criteria weights, W_j:

$$W_j = \frac{1 - S_j}{\sum_j^n (1 - S_j)} \quad \text{for } j = 1, 2, \ldots, n \tag{20}$$

Consequently, S_i solution for rankings can be obtained from:

$$S_i = \sum_{j=1}^{n} W_j S_{ij} \quad \text{for } i = 1, 2, \ldots, m \tag{21}$$

Accordingly, the R_i solution becomes:

$$R_i = max\left[W_j S_{ij}\right] \quad \text{for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n \tag{22}$$

It should be re-emphasized that w_j in (5) and (6) for calculating S_i and R_i in the ordinary VIKOR represent the criteria weights. These criteria weights are usually obtained from another MCDM method, e.g. AHP, in the ordinary use of VIKOR. However, these weights represent the weights of *jnd*s in this study and are used to calculate S_{ij} from (19).

Finally, in step 4, the alternatives are ranked by sorting the values of S and R in increasing order according to the ordinary VIKOR. On the other hand, the normalized $(1 - S_i)$ values provide the preference values of alternatives, P_i.

IV. ANALYSIS AND DISCUSSION

The data used in this paper is taken from the study performed by Karacasu [47] who used the ELECTRE method to determine public opinions on the desired type of public bus operating systems between one run by the local authority and one run by private organizations in Eskişehir, Turkey. Later, Arslan [48] developed a hybrid model using concepts from fuzzy and AHP methods and applied the hybrid model to the part of the data. Further, Arslan [49] applied an extended TOPSIS method which is based on behavioral theories to the same data set.

The data represent the opinions of an expert group who are mostly comprised of academicians from different universities and experts working for various transportation organizations in Turkey. A total of 52 experts stated their preferences over the range of zero to 100 for the two alternatives which are a bus system operated by the municipality (bM) and a bus system operated by a private organization (bP) with respect to 6 criteria that are: comfort of service (CS), way of paying fare (WF), service reliability (SR), timely service (TS), flexibility in service management (FM), and standards of vehicles (SV). It should be mentioned that the purpose of this paper is to also show the applicability of the suggested model to reveal subjective opinions in terms of rankings and preferences. Therefore, how the experts and/or criteria are defined or other important issues that can be faced in this particular decision-making process are not discussed in this paper. Furthermore, it should also be emphasized that the suggested method is not limited to analyzing only two alternatives, rather it can be valid for analyzing any number of alternatives as faced in most real MCDA problems.

In order to clarify the procedure; a subject's ratings for alternatives and criteria as given in Table 1 are used as a numerical example. The computational steps for the example follow the procedure presented in Fig. 1 as explained in the previous section.

TABLE 1. A subject's decision matrices (step 1).

		Criteria					
		CS	WF	SR	TS	FM	SV
Rating Criteria		95	80	85	100	85	95
Rating Alternatives	bM	40	45	55	45	55	45
	bP	70	60	55	75	55	80

As described earlier, for the data used in this study, subjects rate the alternatives and criteria over a range of 0-100 which represents $f_{max} = 100$ and $f_{min} = 0$ in (14) of the Weber-Fechner law. On the other hand, they represent $f_j^* = 100$ and $f_j^- = 0$ (for all criteria $j = 1, 2, \ldots 6$) in the suggested VIKOR. Accordingly, as given in Table 1, one of the subjects rates the alternatives bM and bP with respect to six criteria. For instance, he/she rates 40 and 70 with respect to CS and 45 and 60 with respect to WF for the alternatives bM and bP, respectively. In a similar fashion, he/she also rates the criteria, for instance, he/she assigns 95 for CS and 80 for WF.

There could be many prospects for dividing the universe of discourse (the range of f_j^- and f_j^* for each criterion) into subjectively equal parts. However, from a behavioral point of view, as Simon's bounded rationality [50] suggests, decision-makers have a cognitive limit when they compare things; the number of *jnd*s is, therefore, assumed to be limited and set at 5 ($v = 4$ in (12), (13) and (14)) in this study for

TABLE 2. Thresholds values and jnds weights (step 2).

	$f_j^* = 100$ and $f_j^- = 0$ for all criteria					
f_k(thresholds)	f_0=100	f_1=93.75	f_2=87.5	f_3=75	f_4=50	f_5=0
jnds: Δf_k	6.25	6.25	12.5	25	50	
Jnds w_k	9	7	5	3	1	

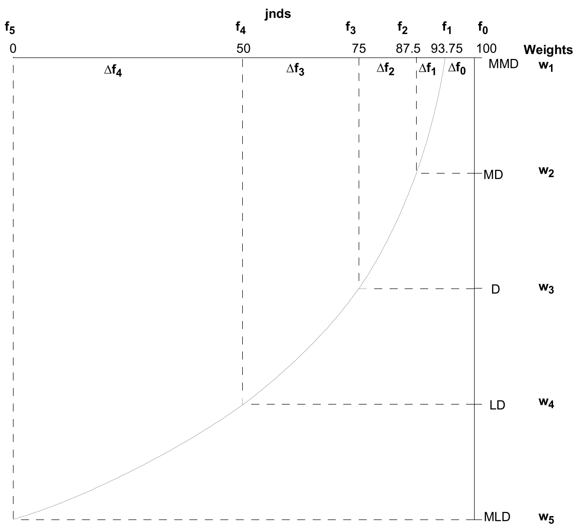

FIGURE 2. Representative jnds and changes in sensation.

ease of computation and manageability. For the data studied, the calculated thresholds and *jnds* are provided in Table 2 and visually displayed in Fig. 2. The abscissa indicates the calculated thresholds or *jnd*s according to (13) and (14) for $(1+\epsilon) = 2$ and $v = 4$, whereas the axis of ordinate shows the corresponding changes in sensation based on the Weber-Fechner law. Moreover, it would also be more meaningful to linguistically label these perceived noticeable distances in the abscissa as 'Much More Desirable' (MMD), 'More Desirable' (MD), 'Desirable' (D), 'Less Desirable' (LD) and 'Much Less Desirable' (MLD) to represent a decision maker's cognitive state corresponding to *jnds*. As recognized, the *jnd*s become narrower as they get closer to the best solution, f_{max} (equivalent to f_j^*) from the worst solution, f_{min} (equivalent to f_j^-). Furthermore, the weights (w_k) of *jnds* given in Table 2 (illustrated on the ordinate in Fig. 2) represent the weights of corresponding *jnd*s as indicating the importance of the intervals to the best solution. The AHP scale (1,3,5,7 and 9), which is commonly used in the decision support field, is in accord with the orders of the judgmental statements used for *jnd*s, and therefore is adopted in this paper. Assuming that the MLD is the base to which the weight 1.0 is assigned, accordingly 3, 5, 7 and 9 are assigned to LD, D, MD and MMD respectively.

Consequently, the criteria weights should be calculated first. Accordingly, S_j are found from (19), for instance $f_1 = 95$ (where; $f_1 \leq f_1 < f_0$) for CS, S_1, as shown at the bottom of this page.

Accordingly, the others are also obtained by following the same procedure ($S_2 = 0.48$, $S_3 = 0.39$, and so on as given in Table 3). Then, criteria weights are calculated by normalizing these $(1 - S_j)$ that are found as $W_1 = 0.19$, $W_2 = 0.12$, $W_3 = 0.14$, $W_4 = 0.23$, $W_5 = 0.14$ and $W_6 = 0.19$ as seen in Table 3.

TABLE 3. Obtained results (step 3 & 4).

		CS	WF	SR	TS	FM	SV
Crt.	S_j	0.16	0.48	0.39	0.00	0.39	0.16
	$1-S_j$	0.84	0.52	0.61	1.00	0.61	0.84
	W_j	0.19	0.12	0.14	0.23	0.14	0.19
Alt.	S_{ij} (bM)	0.86	0.84	0.77	0.84	0.77	0.84
	S_{ij} (bP)	0.62	0.72	0.77	0.57	0.77	0.48
	S_i $(=\sum_{j=1}^{n} W_j S_{ij})$				Ranking		
bM		0.83			2		
bP		0.64			1		
	R_i $(=max[W_j S_{ij}])$				Ranking		
bM		0.19			2		
bP		0.13			1		
			$p_{ij}(=1-S_{ij})$				
	p_{ij} (bM)	0.14	0.16	0.23	0.16	0.23	0.16
	p_{ij} (bP)	0.38	0.28	0.23	0.43	0.23	0.52
	p_i				P_i		
bM		0.18			0.32		
bP		0.54			0.68		

Then, a similar procedure is performed for the alternatives to calculate S_{ij} from (19), for instance $f_{11} = 40$ (where; $f_5 \leq f_{11} < f_4$) for bM with respect to the criterion CS, S_{11}, as shown at the bottom of this page, and $f_{21} = 70$ (where; $f_4 \leq f_{21} < f_3$) for bP, S_{21}, as shown at the bottom of this page. Consequently, the others are obtained ($S_{12} = 0.84$, $S_{22} = 0.72$, $S_{13} = 0.77$, $S_{23} = 0.77$, and so on) in a similar fashion as presented in Table 3.

For ranking, S_i are obtained by multiplying S_{ij} with the associated weights and they are summed up as in (21). Accordingly, the resulting S_i are found to be $S_1 = 0.83$ and $S_2 = 0.64$ indicating that bP is ranked 1[st] and bM is

$$S_1 = \left[\frac{9 \times (100-95)}{1 \times (50-0) + 3 \times (75-50) + 5 \times (87.5-75) + 7 \times (93.75-87.5) + 9 \times (100-93.75)}\right] = 0.16$$

$$S_{11} = \left[\frac{1 \times (50-40) + 3 \times (75-50) + 5 \times (87.5-75) + 7 \times (93.75-87.5) + 9 \times (100-93.75)}{1 \times (50-0) + 3 \times (75-50) + 5 \times (87.5-75) + 7 \times (93.75-87.5) + 9 \times (100-93.75)}\right] = 0.86$$

$$S_{21} = \left[\frac{3 \times (75-70) + 5 \times (87.5-75) + 7 \times (93.75-87.5) + 9 \times (100-93.75)}{1 \times (50-0) + 3 \times (75-50) + 5 \times (87.5-75) + 7 \times (93.75-87.5) + 9 \times (100-93.75)}\right] = 0.62$$

ranked 2nd. According to (22), $R_1 = 0.19$ and $R_2 = 0.13$ result in the same ranking as S_i. Consequently, the preference values can be obtained according to (17) and (18) as provided in Table 3. Simply, p_{ij} (corresponding to $(1 - S_i)$) are multiplied by the associated weights (W_j) and results are summed up and then normalized to determine overall preference values, P_i. As provided in Table 3, the preference values for bM and bP are calculated as 0.32 and 0.68, respectively, which indicate that bP is preferred over bM in this particular case.

As seen, the ratings provided by the subject for the criteria SR and FM are equivalent (85 for SR and FM as seen in Table 1). Moreover, he/she also rates these two criteria equal in importance (55 for the alternatives bM and bP as seen in Table 1), while he/she rates the other criteria (CS, WF, TS, and SV) as clearly in favor of bP. A result favoring bP over bM is therefore obvious. Accordingly, the proposed method assigns a higher preference value to bP (0.68) while 0.32 to bM, which is consistent with the expectation.

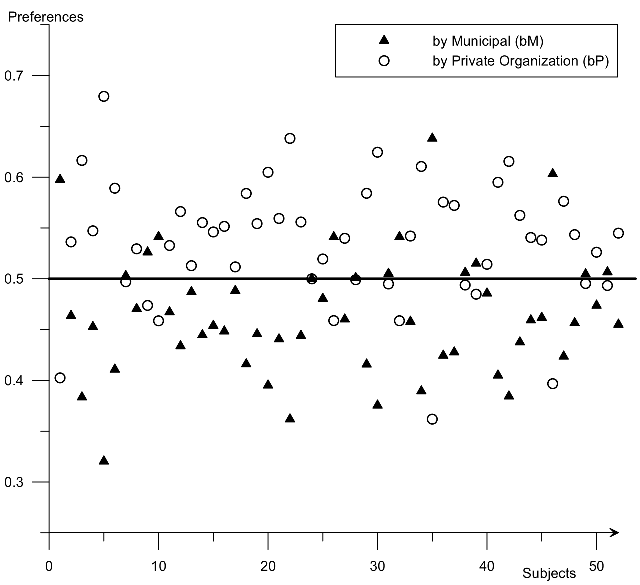

FIGURE 3. **Preference values obtained from the suggested method.**

Following the same procedure, the overall results are obtained and visually displayed in Fig. 3. Since there are two alternatives compared, the horizontal line passing through 0.5 on the ordinate corresponds to the indecision or threshold value over which the selected ones are indicated. As seen, most of the subjects prefer the bus service operated by private organizations over the one operated by the municipality.

Only one subject provided precisely the same ratings for both alternatives concerning the specified criteria (subject 24). Therefore, the method naturally yields one indecisive outcome for this particular subject. Moreover, one of the subjects (subject 49) favors bM by a slim margin. In short, the results show that one subject remains indecisive, 12 subjects prefer bM and 39 subjects prefer bP. It should also be mentioned that most of the subjects rate bM and bP closely for each criterion, therefore the obtained preference results are stuck within the range of 0.30-0.70.

TABLE 4. **Correct classification in terms of rankings.**

		Ordinary MAVT				R			
		bM	bP	Ind	$\sum$	bM	bP	Ind	$\sum$
S	bM	12	-	-	12	3	4	5	12
	bP	-	39	-	39	4	23	13	40
	Ind*	-	-	1	1	-	-	1	1

Note: Ind = indecisive (or bM=bP in rank)

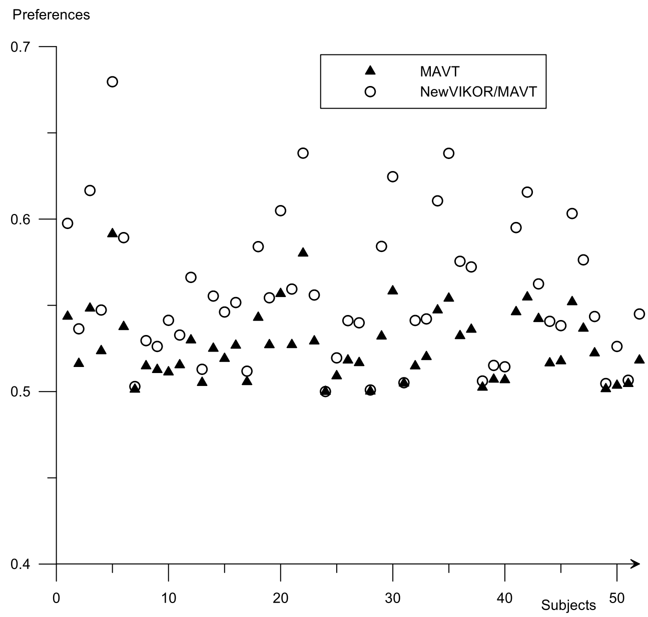

FIGURE 4. **MAVT and new VIKOR/MAVT preferences of preferred alternatives.**

On the other hand, considering the ranking performance, the suggested VIKOR/MAVT approach based on the Weber-Fechner law shows a perfect match with the ordinary MAVT methods as shown in Table 4, where S and R are the rankings obtained from the suggested approach. As explained, R ranking of the ordinary VIKOR is not a proper method for determining subjective judgments since the rankings are defined as the reverse order of $max\left[W_j S_{ij}\right]$ (see (21)). S and R rankings could be the same only if $R_i = S_i$ as mentioned in Section 3 or the order of R_i coincides with the order of S_i in magnitude as in the case given in the example above (see Table 3). Therefore, it is not surprising to see a 44% match between S and R (3 matches for bM, 23 matches for bP and 1 match for indecisive).

Furthermore, in order to see the differences in preferences, the same criteria weights are used for the ordinary MAVT method to obtain choices and for clarification; only the preferred alternatives are compared with the ones obtained from the suggested method as demonstrated in Fig. 4. As seen in Fig. 4, the suggested method inflates the differences, as expected, due to the adopted Weber-Fechner law and the weights used for noticeable differences.

The model is highly non-linear; therefore, it is not useful to consider all the criteria to assess the sensitivity of the

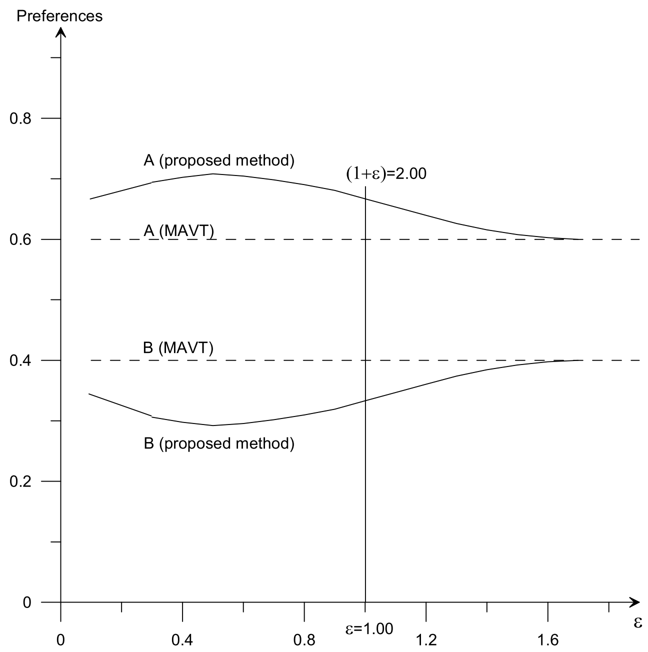

FIGURE 5. **Proposed model sensitivity to variation in progression factor.**

model to *jnd*s variation. Thus, a simple case is considered for performing the sensitivity analysis: two alternatives A and B with ratings of 60 and 40, respectively, are taken into account. The preference values obtained from the suggested method are shown in Fig. 5. The solid lines display the preference values obtained from the proposed method, while the dashed lines indicate the preference values obtained from the MAVT method. The vertical line at $\varepsilon = 1.00$ indicates the progression factor $(1 + \varepsilon) = 2.00$ used in this study as Lootsma [31], [45], [46] suggests (see Section 3 for the discussion). It should be mentioned that the sensitivity of the model outputs to the variation in *jnd*s (or progression factors in (14)) is dependent on locations of the ratings (here 60 and 40 for the simplification) over the range. As seen, the outputs of the proposed method show a non-linear behavior with the variation in progression factors $(1 + \varepsilon)$: as the progression factor increases; the values of A increase and start decreasing at an arbitrary point depending on the location of the ratings on the range (it is around $\varepsilon = 0.5$ for this example). Then, the outputs obtained from the proposed method merge into the preference values obtained from the ordinary MAVT method (again, depending on the location of the ratings on the range; for this example, it is around $\varepsilon = 1.7$). The rankings obtained from the proposed method never contradict the ordinary VIKOR's or MAVT's rankings due to the mathematical formulation. Moreover, the model opens an opportunity to allow a behaviorally plausible model for eliciting subjective preferences to be developed.

As seen, the suggested method is easy to perform and does not bring any burden to those participating in the process since only subjective perceptions are collected on a defined range. Although the proposed model and the ordinary VIKOR provided the same results in terms of ranking, the ordinary VIKOR usually requires another method, i.e. the AHP, to obtain the criteria weights. It is however not required for the proposed method since the proposed model can provide these weights itself by applying the same methodology on criteria. Moreover, results in terms of preferences are more meaningful than rankings since it can allow upper-level decision-makers, e.g., policymakers, to see the general tendency more clearly. In this respect, the suggested method can also allow policymakers to review the variations in preferences with respect to each criterion rather than considering only collective preferences. If needed, this can enable upper-level decision-makers to convince the public and better inform them on particular issues for the necessity of adopting a planned measure.

V. CONCLUSION

As the public's appraisals on transportation projects during the planning process have been gaining importance, integrating behavioral psychology in decision analysis models becomes crucial for pursuing the public's subjective opinions which are mostly based on perceptions. In this paper, the Weber-Fechner law, a well-accepted law in behavioral psychology, is adopted for extending the ordinary VIKOR method for eliciting subjective assessments. The proposed method was applied to a case where two alternatives were evaluated with respect to six different criteria. A numerical example was also provided to illustrate the applicability of the model. As the results obtained from the proposed method and the ordinary MAVT were compared, a total agreement in terms of rankings was observed. However, the preference values of the preferred alternatives were inflated in the proposed methods indicating that the proposed method performs in line with the logarithmic scale of the Weber-Fechner law.

In short, these findings show that the proposed method that is grounded on Weber's law, a basic perceptual principle of psychological science, is easy to apply and can provide intuitive results. Thus, the proposed method is said to be applicable in areas where public opinions based on subjective perceptions including their perceptual discriminations are the main focus. Moreover, considering that subjective preference values are of interest, adopting behavioral psychology into a decision support tool opens up an opportunity to allow behaviorally more realistic model developments that can better explain the human subjective judgment process. In this respect, the proposed method may need further behavioral validation in future work, for instance, examining the variation of *jnd*s among the subjects in a more simple setting.

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