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Journal of Taibah University for Science

ISSN: (Print) 1658-3655 (Online) Journal homepage: https://www.tandfonline.com/loi/tusc20

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To cite this article: Merve Ascioglu & Ismail Naci Cangul (2018) Narumi–Katayama index of the subdivision graphs, Journal of Taibah University for Science, 12:4, 401-408, DOI: 10.1080/16583655.2018.1474542

To link to this article: https://doi.org/10.1080/16583655.2018.1474542

Published online: 21 May 2018.

JOURNAL OF TAIBAH UNIVERSITY FOR SCIENCE
2018, VOL. 12, NO. 4, 401–408
https://doi.org/10.1080/16583655.2018.1474542

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Narumi–Katayama index of the subdivision graphs

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ABSTRACT
Subdivision is an important aspect in graph theory which allows one to calculate properties of some complicated graphs in terms of some easier graphs. Recently, the notion of r-subdivision was similarly defined as a quite useful generalization by adding r new vertices to each edge. Also the double graphs are used to ease some calculations, especially with the chemical graphs. Topological graph indices have become popular due to their applications in chemistry or related areas due to their advantages over time and money consuming laboratory experiments. In this paper, we calculate one of the topological graph indices, namely the Narumi–Katayama index of the subdivision, r-subdivision, double, subdivision of double and double of subdivision graphs of any given graph. We give some symmetry relations and results for the well-known graph classes.

ARTICLE HISTORY
Received 3 April 2018
Accepted 26 April 2018

KEYWORDS
Narumi–Katayama index; topological indices; subdivision graph; r-subdivision graph; double graph; chemical graphs

AMS 2010 SUBJECT CLASSIFICATION NUMBER
05C07; 05C30; 05C38

1. Introduction

Throughout this paper, we assume that all the graphs $G = (V, E)$ under consideration are simple, connected and undirected graphs with the number of vertices $|V(G)| = n$ and the number of edges $|E(G)| = m$. That is, we do not allow loops or multiple edges. For each vertex $v \in V(G)$, the degree of v is denoted by $d_G(v)$ or briefly by $d(v)$. In the special case that a vertex has degree one, this vertex is called a pendant vertex. We shall give our examples for the null, path, cycle, star, complete, complete bipartite and tadpole graphs which are denoted by N_n, P_n, C_n, S_n, K_n, $K_{a,b}$ and $T_{a,b}$, respectively.

There are some fixed invariant numbers which do not change for the isomorphic graphs and gives information about the graph under consideration and these are called topological graph indices. These indices are defined and used in many areas to study several properties of different real life objects such as atoms and molecules in chemistry. These indices can mainly be classified into three groups according to their definitions: those defined by means of matrices, those by means of vertex degrees, and those by means of distances, etc. Some of the most well-known vertex degree-based topological indices are the first and second Zagreb indices, first and second multiplicative Zagreb indices, atom-bond connectivity index, Narumi–Katayama index, geometric-arithmetic indices, harmonic index and sum-connectivity index. In this paper, we particularly deal with the Narumi–Katayama index $NK(G)$ defined as follows:

$$NK(G) = \prod_{u \in V(G)} d_G(u).$$

In the particular case where G is k-regular, we obtain the following result.

$$NK(G) = k^n. \quad (1)$$

Topological graph indices have found many uses in several areas including mainly molecular graph theory due to their advantages over the existing experimental methods. In recent years, a large number of such indices have been defined and utilized for chemical documentation, isomer discrimination, study of molecular complexity, chirality, similarity/dissimilarity, QSAR/QSPR, drug design and database selection, lead optimization, etc. The pharmaceutical industry contributed towards increased interest in molecular descriptors because of the necessity to reduce the expenditure involved in synthesis, in vitro, in vivo, or clinical testing of new medicinal compounds.

This paper is planned as follows: In Section 2, NK index is calculated for some well-known graph classes. In Section 3, NK index of the double graphs were obtained. In Section 4, NK indices of subdivision graphs are calculated. In Section 5, NK index is calculated for the generalised case of subdivision which is called r-subdivision of graphs. In Section 6, some relations between NK indices of several graphs are obtained. Finally in Section 7, NK index of the double graphs of

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subdivision graphs, and in Section 8, NK index of subdivision graphs of the double graphs are found. Although the names of these groups of graphs resemble each other, there are serious differences in the methods of obtaining them. The degree sequence of a graph will be our main aid in combinatorical calculations.

2. Narumi–Katayama index of some graph classes

In this section, to illustrate the results in the following sections, we first calculate the NK indices of some well-known graph classes.

For example for the tadpole graph $T_{4,5}$ in Figure 1, the vertex degrees are 3, 2, 2, 2, 2, 2, 2, 2 and 1. So NK index of $T_{4,5}$ is $3 \cdot 2^7$. In general, for every tadpole graph $T_{a,b}$ it is easy to show that $NK(T_{a,b}) = 3 \cdot 2^{a+b-2}$ as the vertex degrees in $T_{a,b}$ are $3, 2^{(a+b-2)}, 1$. Here $x^{(n)}$ means that x appears n times amongst the degrees.

The NK indices of some frequently-used graph classes are given as follows:

$$NK(G) = \begin{cases} 2^{n-2} & \text{if } G = P_n,\ n \geq 2 \\ 2^n & \text{if } G = C_n,\ n > 2 \\ n-1 & \text{if } G = S_n,\ n \geq 2 \\ (n-1)^n & \text{if } G = K_n,\ n \geq 2 \\ a^b b^a & \text{if } G = K_{a,b},\ a,b \geq 1 \\ 3 \cdot 2^{a+b-2} & \text{if } G = T_{a,b},\ a \geq 3, b \geq 1, \end{cases} \tag{2}$$

see [1]. The proofs of each one follows easily from the definition of NK index combinatorically.

3. Narumi–Katayama index of double graphs

For a graph G with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$, we take another copy of G with vertices labelled by $\{v_1, v_2, \ldots, v_n\}$, this time, where each vertex v_i corresponds to v_i for each i. If we connect all vertices v_i to the neighbours of v_i at the other copy for each i, we obtain a new graph called the double graph of G. It is denoted by $D(G)$ (Figures 2 and 3).

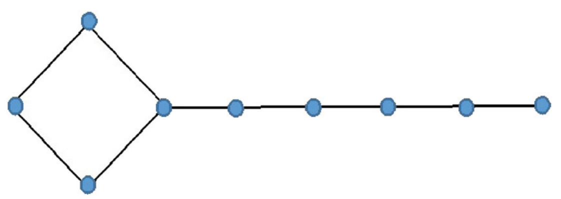

Figure 1. Tadpole graph $T_{4,5}$.

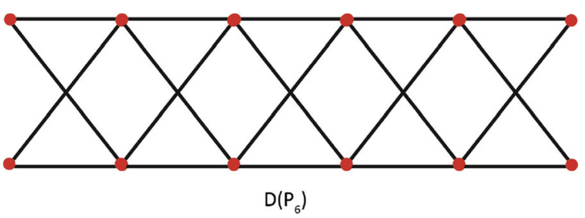

Figure 2. Double graph of P_6.

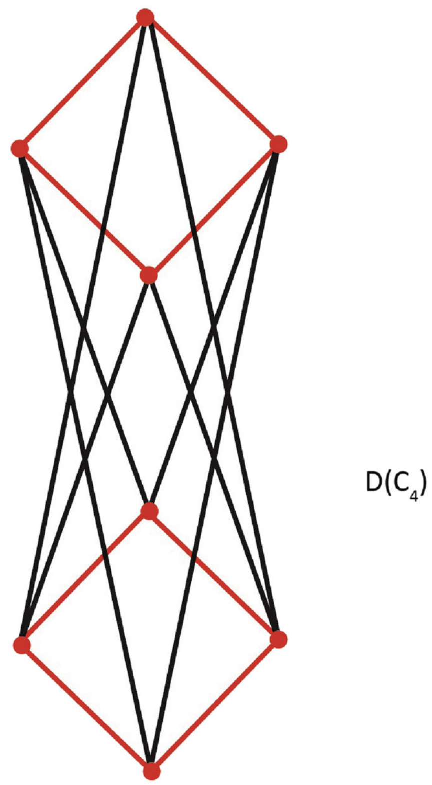

Figure 3. Double graph of C_4.

Double graphs were first introduced by Indulal and Vijayakumar [2] in the study of equienergetic graphs. Later Munarini et al. [3] calculated the double graphs of N_n and $K_{a,b}$ as N_{2n} and $K_{2a,2b}$, respectively.

The notion of degree of a graph provides us an area to study various structural properties of graphs and hence attracts the attention of many graph theorists and also other scientists including chemists. If d_i, $1 \leq i \leq n$, are the degrees of the vertices v_i of a graph G in any order, then the *degree sequence* (*DS*) of G is the sequence $\{d_1, d_2, \ldots, d_n\}$. In some places, it is also denoted by $\{d_1 d_2 \ldots d_n\}$, but we prefer the former notation. Also, in many papers, the DS is taken to be a non-increasing sequence, whenever possible.

Conversely, a non-negative sequence $\{d_1, d_2, \ldots, d_n\}$ will be called *graphical* or *realizable* if it is the DS of any graph. It is clear from the definition that for a graphical DS, there is at least one graph having this DS. For example, the completely different two graphs in Figure 4 have the same DS.

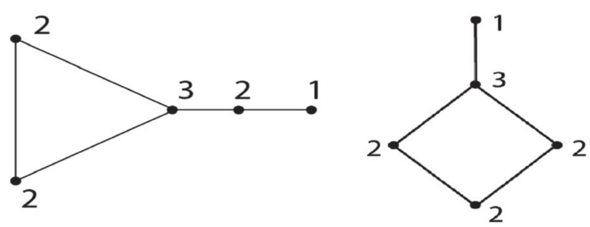

Figure 4. Graphs with the same DS.

For convenience and brevity, we shall denote a DS having repeated degrees with a shorter DS. For example, if the degree d_i of the vertex v_i appears z_i times in the DS of a graph G, then we use $\{d_1^{(z_1)}, d_2^{(z_2)}, \ldots, d_l^{(z_\ell)}\}$ instead of $\{d_1, d_2, \ldots, d_n\}$ where $\ell \leq n$. Here the members z_i are called the *frequencies* of the degrees. When $\ell = n$, that is, when all degrees are different, the DS is called *perfect*.

It is an open problem to determine that which DSs are graphical and there are several algorithms to determine that.

Another important reason to study the DSs of graphs is topological indices mentioned above. Therefore to know about the DS of the graph will help to obtain information about, e.g. the chemical properties of the graph.

The modern study of DSs started in 1981 by Bollobas [4]. The same year, Tyshkevich et al. established a correspondence between DS of a graph and some structural properties of this graph [5]. In 1987, Tychkevich et al. [6] written a survey on the same correspondence . In [7], the authors gave a new version of the Erdös–Gallai theorem on the realizability of a given DS. In 2008, a new criterion on the same problem is given by Triphati and Tyagi [8]. The same year, Kim et al. gave a necessary and sufficient condition for the same problem [9]. Ivanyi et al. [10] gave an enumeration of DSs of simple graphs. Miller [11] also gave a criteria for the realizability of given DSs. But still, the most respected method to determine a DS is graphical is the one given by Hakimi [12] and Havel [13]. Recently, the second author together with Delen and Cangul [14], have defined a new graph invariant which is useful in determining the realizability of a given DS and also gives a large number of properties of the family of the realization graphs.

Let now G be a simple graph. The degree sequence of the double graph of G can be given in terms of the degree sequence of G as follows:

Lemma 3.1 ([15]): *Let the DS of a graph G be* $DS(G) = \{d_1, d_2, \ldots, d_n\}$. *Then the DS of the double graph* $DS(D(G))$ *is given by*

$$DS(D(G)) = \left\{2d_1^{(2)}, 2d_2^{(2)}, \ldots, 2d_n^{(2)}\right\}.$$

Hence the relation between the NK index of G and the NK index of $D(G)$ is given by the following result:

Theorem 3.1: *For any simple graph G, we have*

$$NK(D(G)) = [2^n \cdot NK(G)]^2.$$

Proof: Let the degree sequence of a graph G be $DS(G) = \{d_1, d_2, \ldots, d_n\}$. Then by Lemma 3.1, we have

$$\begin{aligned} NK(D(G)) &= (2d_1)^2 (2d_2)^2 \ldots (2d_n)^2 \\ &= [2^n \cdot NK(G)]^2, \end{aligned}$$

as required. ■

4. Narumi–Katayama index of subdivision graphs

The subdivision graph $S(G)$ of a graph G is the graph obtained from G by replacing each of its edges by a path of length 2, or equivalently by inserting an additional vertex into each edge of G. The subdivision graph of the cycle graph is illustrated in Figure 5.

Subdivision graphs are used to obtain several mathematical and chemical properties of more complex graphs from more basic graphs and there are many results on these graphs, see Figure 6.

There are several papers dealing with some topological indices, mainly several Zagreb indices and coindices, of the subdivision graphs of some graphs. In [16], the Zagreb indices of the line graphs of the subdivision graphs were studied. In [17], Zagreb indices of the subdivision graphs were calculated. In [18], all 10 versions of Zagreb indices and coindices of subdivision graphs of certain graph types were calculated. Similarly, in [15], Zagreb indices and multiplicative Zagreb indices of subdivision graphs of double graphs which are obtained by taking another copy of the given graph and joining all vertices in one to all neighbour vertices in the other were found. In [19], another use of subdivision graphs is introduced by studying the total domination subdivision number of any graph which is equal to the minimum number of edges of the graph that must be subdivided in order to increase the total domination number.

Let now G be a simple graph. The relation between the NK index of G and NK index of $S(G)$ is given by the following result:

Theorem 4.1: *For any simple graph G, we have*

$$NK(S(G)) = 2^m \cdot NK(G).$$

Proof: Recall that G has m edges. To obtain the subdivision graph $S(G)$, we add a new vertex to each edge

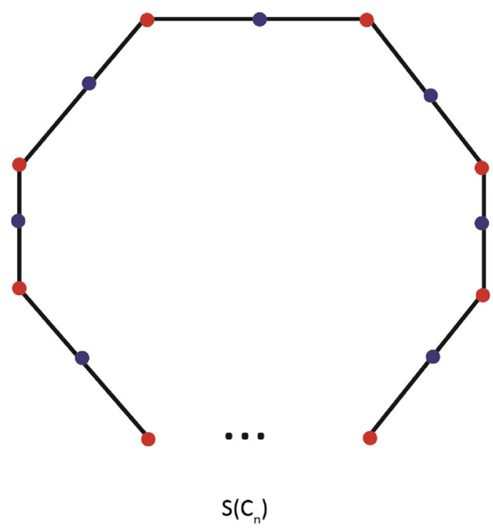

Figure 5. Subdivision of the cycle graph C_n.

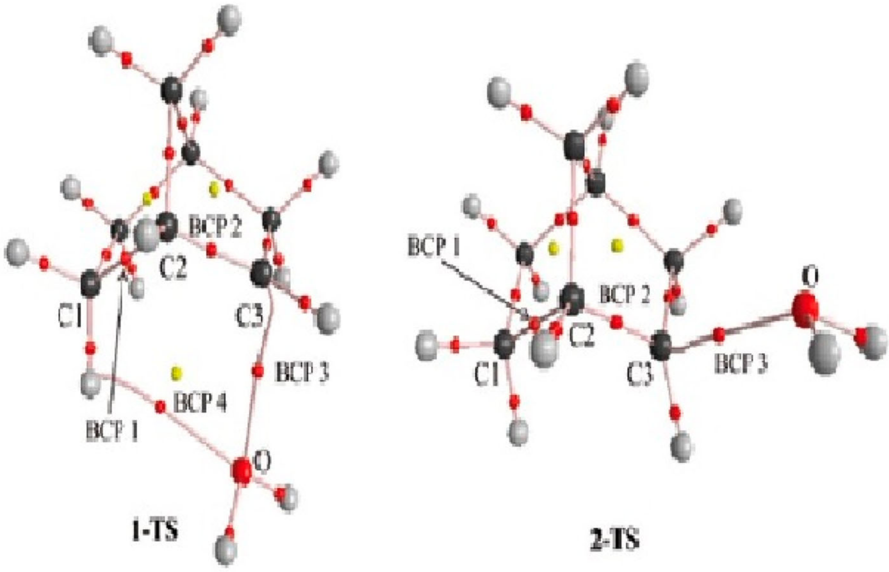

Figure 6. Molecular use of subdivision graphs.

which is of degree 2, and this operation does not effect the other vertex degrees. Hence the result follows. ■

By means of Theorem 4.1, one can easily compute the NK index of the subdivision graphs of the above graph classes.

5. Narumi–Katayama index of *r*-subdivision graphs

Similarly the *r*-subdivision graph of G denoted by $S^r(G)$ was defined by adding r vertices to each edge by Togan et al. [18], and their Zagreb indices and coindices were obtained.

In [20], some formulae and inequalities on several Zagreb indices of *r*-subdivision graphs were obtained. In [21], the authors studied the *r*-subdivision graphs of double graphs and their multiplicative Zagreb indices. In this paper we shall calculate the *NK* index of all these graphs. We also give some relations between these numbers. The double graphs of the subdivision graphs were studied in [22]. These subdivision and *r*-subdivision graphs were recently studied by Lokesha, Togan, Yurttas and Cangul [18].

Similarly to Theorem 4.1, we can obtain the NK index of the *r*-subdivision graphs of any simple graph G:

Theorem 5.1: *For any simple graph G, we have*

$$NK(S^r(G)) = 2^{r\cdot|E(G)|} \cdot NK(G) = 2^{rm} \cdot NK(G).$$

Proof: Recall that G has m edges and as each of these m edges has r new vertices of degree 2 each in $S^r(G)$, we get the required result. ■

One can give the relation between NK index of $S^r(G)$ and NK index of $S(G)$ as below.

Corollary 5.1: *For any simple graph G, we have*

$$NK(S^r(G)) = 2^{m(r-1)} \cdot NK(S(G)).$$

The proof follows directly from the above results.

If the graph under consideration is regular, then we can state our result more clearly:

Corollary 5.2: *For any simple regular graph G, we have*

$$NK(S^r(G)) = 2^{rm} \cdot NK(G).$$

Using Equation (1), we can state this formula in a different form if the graph is k-regular:

$$NK(S^r(G)) = 2^{rm} \cdot k^n.$$

Finally in this section, we calculate the NK index of the *r*-subdivision graphs of some well-known graph classes in terms of the number n of the vertices of G and r:

$$NK(S^r(G)) = \begin{cases} 2^{n-2+r(n-1)} & \text{if } G = P_n,\ n \geq 2 \\ 2^{n(r+1)} & \text{if } G = C_n,\ n > 2 \\ (n-1)2^{r(n-1)} & \text{if } G = S_n,\ n \geq 2 \\ (n-1)^n 2^{rn(n-1)/2} & \text{if } G = K_n,\ n \geq 2 \\ 2^{r(a+b)} a^b b^a & \text{if } G = K_{a,b},\ a, b \geq 1 \\ 3 \cdot 2^{(r+1)(a+b)-2} & \text{if } G = T_{a,b},\ a \geq 3, b \geq 1. \end{cases} \quad (3)$$

6. Relations between NK indices of *r*-subdivision graphs of several graph types

In this section, our aim is to obtain some relations giving the NK indices of the above graph classes in terms of the NK indices of some easier graphs. First, note that if G is a path, cycle or tadpole graph, then the subdivision and r-subdivision of G are also a path, cycle or tadpole graph, respectively. But for other graph classes, the type of the subdivision and r-subdivision of G changes, and the obtained graphs cannot be a member of one of these well-known graph classes. In those cases, we give relations giving the NK indices of these three types of r-subdivision graphs in terms of the NK indices of some other r-subdivision graphs of some other graph classes.

As we have already formulated the NK indices of the r-subdivision graphs of six well-known graph classes in terms of the number n of the vertices of the graph G and the number r, we now reformulate them in terms of the number of vertices n, the number of edges m of the graph G, and the number r. First, let us consider the r-subdivision of the path graph P_n. The obtained graph is again a path graph P_{n+mr}. Therefore by Equation (2),

$$NK(S^r(P_n)) = NK(P_{n+mr}).$$

Hence

$$NK(S^r(P_n)) = 2^{n+mr-2}.$$

Secondly, let us consider the r-subdivision of the cycle graph C_n. The obtained graph is again a cycle graph C_{n+mr}. Similarly, by Equation (2),

$$NK(S^r(C_n)) = NK(C_{n+mr}).$$

Therefore we get

$$NK(S^r(C_n)) = 2^{n+mr}.$$

Let us now consider the r-subdivision of the tadpole graph $T_{a,b}$. As we mentioned above, the obtained graph is again a tadpole graph $T_{a(r+1),b(r+1)}$. This graph can also be stated as $T_{a+mr,b}$ or $T_{a,b+mr}$. Therefore by Equation (2),

$$\begin{aligned} NK(S^r(T_{a,b})) &= NK(T_{a(r+1),b(r+1)}) \\ &= NK(T_{a+mr,b}) \\ &= NK(T_{a,b+mr}). \end{aligned}$$

As a result, we have

$$\begin{aligned} NK(S^r(T_{a,b})) &= 3 \cdot 2^{(a+b)(r+1)-2} \\ &= 3 \cdot 2^{a+b+mr-2}. \end{aligned}$$

Summarizing the above results, we can give

Corollary 6.1: *The NK index of the r-subdivision graph of path, cycle and tadpole graphs are given by*

(i) $NK(S^r(P_n)) = 2^{n+mr-2}$.

(ii) $NK(S^r(C_n)) = 2^{n+mr}$.

(iii) $NK(S^r(T_{a,b})) = 3 \cdot 2^{(a+b)(r+1)-2} = 3 \cdot 2^{a+b+mr-2}$.

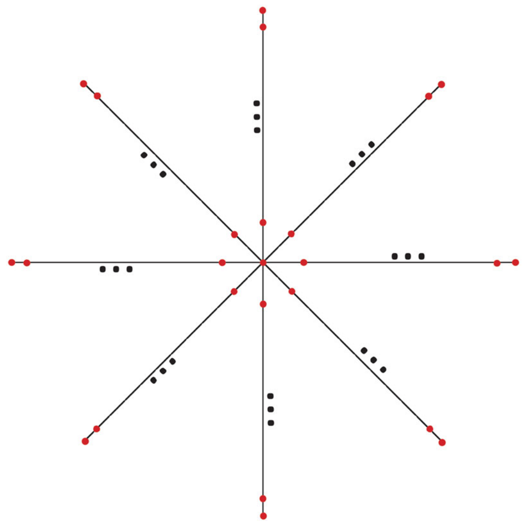

Figure 7. r-subdivision of the star graph S_n.

As we mentioned above, these three types of special graphs are the only graph classes for which the subdivision and r-subdivision of them remain in the same class. Now we shall consider the other three classes of well-known graphs.

The r-subdivision graph of the star graph S_n is shown in Figure 7. We have the following relations:

$$\begin{aligned} NK(S^r(S_n)) &= 2^{mr}(n-1) \\ &= 2^{mr} NK(S_n) \\ &= NK(P_{mr+2}) NK(S_n) \\ &= NK(C_{mr}) NK(S_n). \end{aligned}$$

Now consider the complete graph case. Similarly proceeding, we obtain

Theorem 6.1: *The NK index of the r-subdivision graph of the complete graph is*

$$\begin{aligned} NK(S^r(K_n)) &= NK(K_n) NK(P_{mr+2}) \\ &= NK(K_n) NK(C_{mr}). \end{aligned}$$

Proof: By Equation (3), we have the formula for NK index of the r-subdivision of the complete graph K_n. Replacing $n(n-1)/2$ by m, after some calculations, we obtain the required results. ∎

Finally, we consider the r-subdivision of the complete bipartite graph $K_{a,b}$. We have

Theorem 6.2: *The NK index of the r-subdivision of the complete bipartite graph $K_{a,b}$ is given by*

$$\begin{aligned} NK(S^r(K_{a,b})) &= NK(K_{a,b}) NK(P_{mr+2}) \\ &= NK(K_{a,b}) NK(C_{mr}), \end{aligned}$$

where $m = a+b$.

7. Narumi–Katayama index of the double graphs of subdivision graphs

In Figure 7, we see the double graph of the subdivision of the cycle graph C_3. To calculate the NK index of the double graphs of the subdivision graphs, we need the following lemmas (Figure 8):

Lemma 7.1: *Let $|V(G)| = n$ and $|E(G)| = m$. Then*

$$|V(S(G))| = n + m,$$
$$|E(S(G))| = 2m,$$
$$|V(D(S(G)))| = 2|V(S(G))| = 2n + 2m$$

and

$$|E(D(S(G)))| = 8m.$$

The following result gives the DS of the subdivision graph and the double graph of the subdivision graph for any graph G:

Lemma 7.2: *Let the DS of a graph G be $DS(G) = \{d_1, d_2, \ldots, d_n\}$. Then the DS of the subdivision graph $DS(S(G))$ and the double graph $DS(D(S(G)))$ of the subdivision graph are given by*

$$DS(S(G)) = \left\{d_1, d_2, \ldots, d_n, 2^{(m)}\right\}$$

and

$$DS(D(S(G))) = \left\{2d_1^{(2)}, 2d_2^{(2)}, \ldots, 2d_n^{(2)}, 4^{(2m)}\right\}.$$

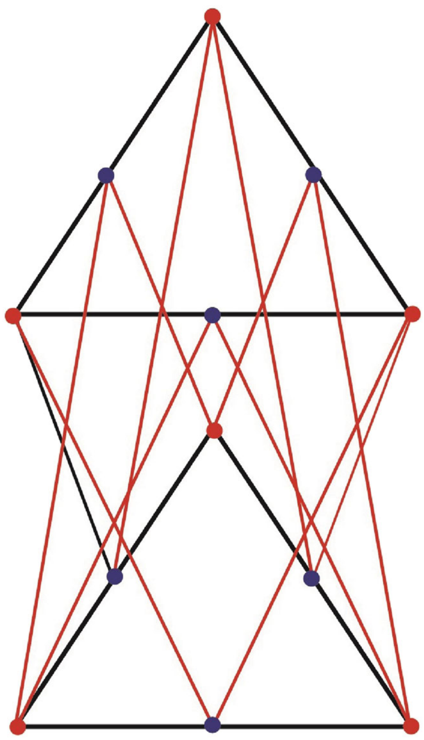

Figure 8. Double graph of the subdivision of C_3.

The next theorem which can now easily be obtained from the earlier results gives the relation between the NK index of the double graph of the subdivision graph and the NK index of the graph:

Theorem 7.1: $NK(D(S(G))) = 2^{4m+2n} \cdot [NK(G)]^2.$

The order and size of the subdivision graph of the double graphs of some well-known graph classes are given in the following example:

Example 7.1:

$$|E(D(S(G)))| = \begin{cases} 8n-8 & \text{if } G = P_n,\ n \geq 2 \\ 8n & \text{if } G = C_n,\ n > 2 \\ 8n-8 & \text{if } G = S_n,\ n \geq 2 \\ 4n(n-1) & \text{if } G = K_n,\ n \geq 2 \\ 8ab & \text{if } G = K_{a,b},\ a,b \geq 1 \\ 8a+8b & \text{if } G = T_{a,b},\ a \geq 3, b \geq 1. \end{cases}$$

Now we can calculate the NK indices of the above well-known graph classes:

Corollary 7.1: *The NK indices of the double of the subdivision graphs of some well-known graph classes are*

$$NK(D(S(G))) = \begin{cases} 2^{4n+4m-4} & \text{if } G = P_n,\ n \geq 2 \\ 2^{4n+4m} & \text{if } G = C_n,\ n > 2 \\ 2^{2n+4m} \cdot (n-1)^2 & \text{if } G = S_n,\ n \geq 2 \\ 2^{2n+4m} \cdot (n-1)^{2n} & \text{if } G = K_n,\ n \geq 2 \\ a^{2b} \cdot b^{2a} \cdot 2^{4ab+2a+2b} & \text{if } G = K_{a,b},\ a,b \geq 1 \\ 3^2 \cdot 2^{8a+8b-4} & \text{if } G = T_{a,b},\ a \geq 3, b \geq 1. \end{cases}$$

8. Narumi–Katayama index of subdivision graphs of the double graphs

In this section, we shall calculate the NK index of subdivision graph of the double graph of any general graph. Our main tool is the degree sequence of a graph. We formulate this index in terms of the NK index of G for a general graph and then calculate it for the six graph classes as a result. We finally note that for complete, tadpole and complete bipartite graphs, the order of double and subdivision does not matter when the NK index is calculated. First we have

Lemma 8.1: *Let G has m edges. Then we have the number of the vertices and the number of the edges of the subdivision graph $S(D(G))$ of the double graph of G are given by*

$$|V(S(D(G)))| = |V(D(G))| + |E(D(G))|$$

and

$$|E(S(D(G)))| = 8m.$$

Hence, the DSs of the double graph $D(G)$ and the subdivision graph $S(D(G))$ of the double graph of G are given by the following result:

Lemma 8.2: *Let the DS of a graph G be* $DS(G) = \{d_1, d_2, \ldots, d_n\}$. *Then the DS of the double graph* $DS(D(G))$ *and the subdivision graph* $DS(S(D(G)))$ *of the double graph are given by*

$$DS(D(G)) = \left\{2d_1^{(2)}, 2d_2^{(2)}, \ldots, 2d_n^{(2)},\right\}$$

and

$$DS(S(D(G))) = \left\{2d_1^{(2)}, 2d_2^{(2)}, \ldots, 2d_n^{(2)}, 2^{(|E(D(G))|)}\right\}.$$

It is now possible to calculate the NK index of the subdivision graph $DS(S(D(G)))$ of the double graph of any graph G:

Theorem 8.1: *For any graph G, the NK index of the subdivision graph* $DS(S(D(G)))$ *of the double graph is* $NK(S(D(G))) = 2^{|E(D(G))|} \cdot [2^n \cdot NK(G)]^2$.

For the above classes of graphs, the NK indices are given by

Corollary 8.1: *The NK indices of the subdivision of the double graph of some well-known graphs are given by*

$$NK(S(D(G))) = \begin{cases} 2^{6n+2m-6} & \text{if } G = P_n,\ n \geq 2 \\ 2^{6n+2m} & \text{if } G = C_n,\ n > 2 \\ 2^{4n+2m-2} \cdot (n-1)^2 & \text{if } G = S_n,\ n \geq 2 \\ 2^{2n+4m} \cdot (n-1)^{2n} & \text{if } G = K_n,\ n \geq 2 \\ a^{2b} \cdot b^{2a} \cdot 2^{4ab+2a+2b} & \text{if } G = K_{a,b},\ a, b \geq 1 \\ 3^2 \cdot 2^{8a+8b-4} & \text{if } G = T_{a,b},\ a \geq 3, b \geq 1. \end{cases}$$

We finally have a nice symmetry property for the NK index of some graphs:

Corollary 8.2: *The NK index of the double graph of the subdivision graph is equal to the NK index of the subdivision of the double graph for the complete, tadpole and complete bipartite graphs. That is*

$$NK(D(S(G))) = NK(S(D(G)))$$

for the complete, tadpole and complete bipartite graphs.

9. Summary and conclusion

In the last few decades, applications of graphs rapidly increase and each day, a method of applying graphs to a new area is found. One of the general ways of studying with graphs for their applications is to study the graph indices which are just some mathematical formulae that give us a number for each graph which is invariant for isomorphic graphs. By using this number, we can obtain several social, physical, chemical, etc. properties of the object which is modelled by the graph. For example, if a graph is used to model a chemical molecule, then such graph indices might help us in determining the melting and boiling points, atomic weight, branchedness, etc and also in QSAR/QSPR studies.

In this paper, one of the fundamental graph indices called Narumi–Katayama index is studied. We calculated this index for several well-known graph classes first to obtain some feeling of what it does, and afterwards, we found it for double graphs, subdivision graphs, *r*-subdivision graphs, and their mixed types as these kind of derived graphs help us to obtain information on larger graphs by means of smaller graphs which need less calculation.

We compared the obtained results for different graph classes and think that the results obtained here will be useful to those studying graph theory and especially for chemists and pharmacologists.

Disclosure statement

No potential conflict of interest was reported by the authors.

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